

Probability and Information Retrieval

**Introduction to Artificial Intelligence
COS302
Michael L. Littman
Fall 2001**

Administration

Foundations of Statistical Natural Language Processing

By Christopher D. Manning and Hinrich Schütze

Grade distributions online.

The IR Problem

query

- doc1
- doc2
- doc3

...

Sort docs in order of relevance to query.

Example Query

Query: The 1929 World Series

384,945,633 results in Alta Vista

- **GNU's Not Unix! - the GNU Project and the Free Software Foundation (FSF)**
- **Yahoo! Singapore**
- **The USGenWeb Project - Home Page**
- ...

Better List (Google)

- **TSN Archives: The 1929 World Series**
- **Baseball Almanac - World Series Menu**
- **1929 World Series - PHA vs. CHC - Baseball-Reference.com**
- **World Series Winners (1903-1929) (Baseball World)**

Goal

Should return as many relevant docs as possible
recall

Should return as few irrelevant docs as possible
precision

Typically a tradeoff...

Main Insights

How identify "good" docs?

- More words in common is good.
- Rare words more important than common words.
- Long documents carry less weight, all other things being equal.

Bag of Words Model

Just pay attention to which words appear in document and query.
Ignore order.



Boolean IR

"and" all uncommon words

Most web search engines.

- Altavista: 79,628 hits
- fast
- not so accurate by itself

Example: Biography

Science and the Modern World (1925), a series of lectures given in the United States, served as an introduction to his later metaphysics. Whitehead's most important book, *Process and Reality* (1929), took this theory to a level of even greater generality.

<http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Whitehead.html>

Vector-space Model

For each word in common between document and query, compute a weight. Sum the weights.

tf = (term frequency) number of times term appears in the document

idf = (inverse document frequency) divide by number of times term appears in any document

Also various forms of document-length normalization.

Example Formula

i	sum _j tf _{i,j}	df _i
Insurance	10440	3997
Try	10422	8760

Weight(i,j) = (1+log(tf_{i,j})) log N/df_i

Unless tf_{i,j} = 0 (then 0).

N documents, df_i doc frequency

Cosine Normalization

$$\text{Cos}(q,d) = \frac{\sum_i q_i d_i}{\sqrt{\sum_i q_i^2} \sqrt{\sum_i d_i^2}}$$

Downweights long documents.
(Perhaps too much.)

Probabilistic Approach

Lots of work studying different weighting schemes.

Often very *ad hoc*, empirically motivated.

Is there an analog of A^* for IR?
Elegant, simple, effective?

Language Models

Probability theory is gaining popularity. Originally speech recognition:

If we can assign probabilities to sentence and phonemes, we can choose the sentence that minimizes the chance that we're wrong...

Probability Basics

$\text{Pr}(A)$: Probability A is true

$\text{Pr}(AB)$: Prob. both A & B are true

$\text{Pr}(\sim A)$: Prob. of not A: $1 - \text{Pr}(A)$

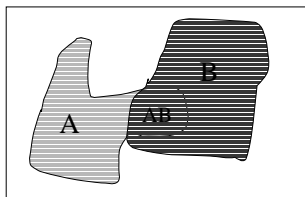
$\text{Pr}(A|B)$: Prob. of A given B

$$\frac{\text{Pr}(AB)}{\text{Pr}(B)}$$

$\text{Pr}(A+B)$: Probability A or B is true

$$\text{Pr}(A) + \text{Pr}(B) - \text{Pr}(AB)$$

Venn Diagram



Bayes Rule

$$\text{Pr}(A|B) = \frac{\text{Pr}(B|A) \text{Pr}(A)}{\text{Pr}(B)}$$

because

$$\text{Pr}(AB) = \text{Pr}(B) \text{Pr}(A|B) = \text{Pr}(B|A) \text{Pr}(A)$$

The most basic form of "learning":

- picking a likely model given the data
- adjusting beliefs in light of new evidence

Probability Cheat Sheet

Chain rule:

$$\Pr(A, X|Y) = \Pr(A|Y) \Pr(X|A, Y)$$

Summation rule:

$$\Pr(X|Y) = \Pr(A \text{ X } | Y) + \Pr(\sim A \text{ X } | Y)$$

Bayes rule:

$$\Pr(A|BX) = \Pr(B|AX) \Pr(A|X) / \Pr(B|X)$$

Speech Example

$$\Pr(\text{sentence}|\text{phonemes})$$

$$= \Pr(\text{phonemes}|\text{sentence})$$

$$\Pr(\text{sentence}) / \Pr(\text{phonemes})$$

Constant

Pronunciation model

Language model

Classification Example

Given a song title, guess if it's a country song or a rap song.

- U Got it Bad
- Cowboy Take Me Away
- Feelin' on Yo Booty
- When God-Fearin' Women Get The Blues
- God Bless the USA
- Ballin' out of Control

Probabilistic Classification

Language model gives:

- $\Pr(T|R)$, $\Pr(T|C)$, $\Pr(C)$, $\Pr(R)$

Compare

- $\Pr(R|T)$ vs. $\Pr(C|T)$
- $\Pr(T|R) \Pr(R) / \Pr(T)$ vs. $\Pr(T|C) \Pr(C) / \Pr(T)$
- $\Pr(T|R) \Pr(R)$ vs. $\Pr(T|C) \Pr(C)$

Naïve Bayes

$$\Pr(T|C)$$

Generate words independently

$$\Pr(w_1 w_2 w_3 \dots w_n|C)$$

$$= \Pr(w_1|C) \Pr(w_2|C) \dots \Pr(w_n|C)$$

$$\text{So, } \Pr(\text{party}|R) = 0.02, \\ \Pr(\text{party}|C) = 0.001$$

Estimating Naïve Bayes

Where would these numbers come from?

Take a list of country song titles.

First attempt:

$$\Pr(w|C) = \text{count}(w; C) \\ / \sum_w \text{count}(w; C)$$

Smoothing

Problem: Unseen words.

$$\Pr(\text{party}|\text{C}) = 0$$

Pr(Even Party Cowboys Get the Blues) = 0

Laplace Smoothing:

$$\Pr(w|\text{C}) = \frac{(1+\text{count}(w; \text{C}))}{\sum_w (1+\text{count}(w; \text{C}))}$$

Other Applications

Filtering

- **Advisories**

Text classification

- **Spam vs. important**
- **Web hierarchy**
- **Shakespeare vs. Jefferson**
- **French vs. English**

IR Example

$$\Pr(d|q) = \frac{\Pr(q|d)\Pr(d)}{\Pr(q)}$$

Language model

Constant

Prior belief d is relevant
(assume equal)

Can view each document like a category for classification.

Smoothing Matters

$$p(w|d) =$$

$p_s(w|d)$ if $\text{count}(w;d) > 0$ (seen)

$p(w|\text{collection})$ if $\text{count}(w;d) = 0$

$p_s(w|d)$: estimated from document and smoothed

$p(w|\text{collection})$: estimated from corpus and smoothed

Equivalent effect to TF-IDF.

What to Learn

IR problem and TF-IDF.

Unigram language models.

Naïve Bayes and simple Bayesian classification.

Need for smoothing.

Homework 6 (due 11/14)

1. Use the web to find sentences to support the analogy **traffic:street::water:riverbed**. Give the sentences and their sources.
2. Two common Boolean operators in IR are "and" and "or". (a) Which would you choose to improve recall? (b) Which would you use to improve precision?

Homework 6 (cont'd)

3. Argue that the language modeling approach to IR gives an effect like TF-IDF. (a) First, argue that $\Pr(q|d) > \Pr(q'|d)$ if q' is just like q but