Games of Chance

Introduction to
Artificial Intelligence

COS302

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Administration

Rush hour (10/22).

Today not part of midterm (10/24), just final.

Uncertainty in Search

We've assumed everything is known: starting state, neighbors, goals, etc.

Often need to make decisions even though some things are uncertain.

Complicates things...

Types of Uncertainty

Opponent: What will other player do?

- Minimax

Outcome: Which neighbor get?

- Model via probability distribution

State: Where are we now?

- Hidden information

Transition: What are the rules?

- Need to use learning to find out

Nim-Rand

Pile of sticks.

- Lose if take last stick.
- On your turn, take 1 or 2.
- Flip a coin. If H, take 1 more.

Which type of uncertainty?

Value of a Game

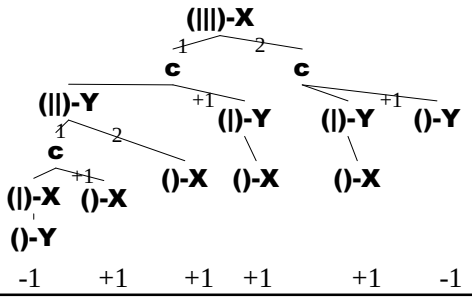
Without randomness: maximize your winnings in the worst case.

With randomness: maximize your *expected* winnings in the worst case.

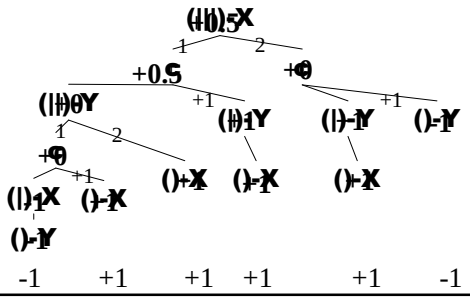
Want to do well on average.

What games are like this?

Nim-Rand Tree



Nim-Rand Values



Search Model

- States, terminal states (G), values for terminal states (V).
- X states (maximizer), Y states (minimizer), Z states (chance)
- For all s in Z, for all s' in N(s)
  - P(s'|s) is the probability of reaching s' from s.

Game Value (no loops)

```
Gameval(s) = {
If (G(s)) return V(s)
Else if s in X
  return max_{s' in N(s)} Gameval(s')
Else if s in Y
  return min_{s' in N(s)} Gameval(s')
Else
  return sum_{s' in N(s)} P(s'|s) Gameval(s')
}
```

Games with Loops

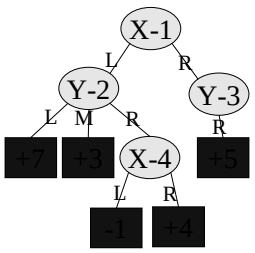
- No known poly time algorithm.
- Approximated by *value iteration*:
- For all s, if G(s), L(s) = V(s), else 0
- Repeat until changes are small:
  - for all s, L(s) =
    - max, min, avg L(s'), s' in N(s)

depending on s in X, Y, or Z.

Hidden Information

- Games like Poker, 2-player bridge, Scrabble ™, Diplomacy, Stratego
- Don't fit game tree model, even when chance nodes included.

Pure Strategies



X: I: 1=L, 4=L
II: 1=L, 4=R
III: 1=R, 4=L
IV: 1=R, 4=R

Y: I: 2=L, 3=R
II: 2=M, 3=R
III: 2=R, 3=R

Matrix Form

Summarizes all decisions in one for each, chosen simultaneously

	X-I	X-II	X-III	X-IV
Y-I	7	7	2	2
Y-II	3	3	2	2
Y-III	-1	4	2	2

Value of Matrix Game

X picks column with largest min

Y picks row with smallest max

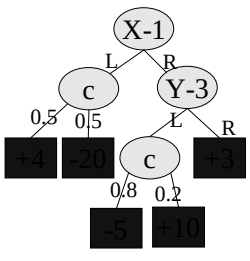
	X-I	X-II	X-III	X-IV
Y-I	7	7	2	2
Y-II	3	3	2	2
Y-III	-1	4	2	2

Minimax

Von Neumann proved zero-sum matrix game, minimax=maximin.

Given perfect information (no state uncertainty), there exists optimal pure strategy for each player.

Game w/ Chance Nodes



Use expected values

	X-I (L)	X-II (R)
Y-I (L)	-8	-2
Y-II (R)	-8	+3

More General Matrices

What game tree leads to this matrix?

Does von Neumann's theorem still hold?

	X-I (L)	X-II (R)
Y-I (L)	1	0
Y-II (R)	0	1

Hidden Info. Matrices

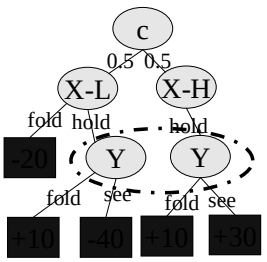
X picks L or R, keeping the choice hidden from Y.

Y makes a choice.

X's choice is revealed and game ends.

	X-I (L)	X-II (R)
Y-I (L)	1	0
Y-II (R)	0	1

Micro Poker



X is dealt high or low card, holds/folds.

Y folds/sees.

High card wins

Y can't see X's card.

Matrix Form

	X-I (fold)	X-II (hold)
Y-I (fold)	-5	+10
Y-II (see)	+5	-5

Player X can guarantee itself +1 on average. How?

It can even announce its strategy.

Mixed Strategies

Pick a number p.

X: With prob. p, fold; else hold.

Since Y doesn't know what's coming, the response will sometimes work, sometimes not.

Guess a Probability

X announces p=1/3.

Y's pick?

	X-I (fold)	X-II (hold)
Y-I (fold)	-5	+10
Y-II (see)	+5	-5

Fold: +5

See: -1 2/3

see

Guess a Probability

X announces p=2/3.

Y's pick?

	X-I (fold)	X-II (hold)
Y-I (fold)	-5	+10
Y-II (see)	+5	-5

Fold: +0

See: +1 2/3

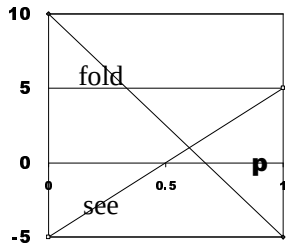
fold

All Strategies

What should X pick for p to maximize its worst case?

p=0.6

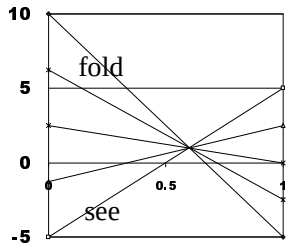
Payoff +1



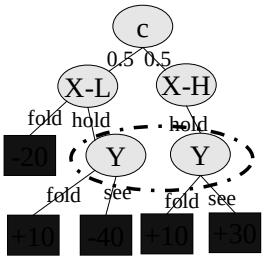
Randomizing Y

If Y random, answer is the same.

No matter what, X can guarantee itself +1.



Bluffing



X: On a low card, bluff with prob. 0.4.

Y: On hold, fold with prob. 0.4.

Solving 2x2 Game

X-I with prob. p

X's expected gain vs. Y-I :

$m_{11}p+m_{12}(1-p)$

vs. Y-II :

$m_{21}p+m_{22}(1-p)$

Maximize the minimum.

	X-I	X-II
Y-I	m_{11}	m_{12}
Y-II	m_{21}	m_{22}

Try p=0, p=1, where lines meet.

Solving General mxn

Linear program: $p_1,\ldots,p_n$.

$p_1+\ldots+p_n = 1$, $p_i \geq 0$

Maximize X's gain, g

vs Y-I: $m_{11}\, p_1 + \ldots + m_{n1}\, p_n \geq g$

vs Y-II: $m_{12}\, p_1 + \ldots + m_{n2}\, p_n \geq g$

...

Against all Y strategies.

Issues

Can we solve poker?

- **More than 2 players**
- **Not zero sum (collude)**
- **Huge state space**

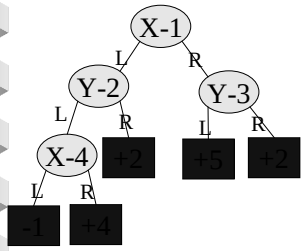
Poker: Opponent modeling

Bridge: Use simulation to approximate

What to Learn

- **Minimax value in games of chance and the DFS algorithm for computing it.**
- **Converting games to matrix form.**
- **Solve 2x2 game.**

Homework 5 (due 11/7)

1. **The value iteration algorithm from the *Games of Chance* lecture can be applied to deterministic games with loops. Argue that it produces the same answer as the "Loopy" algorithm from the *Game Tree* lecture.**
2. **Write the matrix form of the game tree below.**

Game Tree