COS 487: Theory of Computation **Fall 2000**

Mid-Term Exam

Due: Within 3 days *Sanjeev Arora*

Instructions: This exam has five questions, some of which have subparts. Finish the test within **24 hours** after first reading it. Questions 6 and 7 are optional, for which you can take another 12 hours. Write answers legibly in the space provided. If you need extra space for an answer, you may attach extra sheets, *which the instructor will read at his discretion.* You can consult any notes/handouts from this class as well as the text. Feel free to quote, without proof, any results from class or the text. You cannot consult any other source or person in any way.

DO NOT READ THE TEST BEFORE YOU ARE READY TO WORK ON IT.

Your Name:

Write and sign the honor code pledge[1] here:

Prof. Arora will try to answer all emails about the exam promptly. You can also try to reach him between 10am and 10pm at 683-1257. In case of unresolved doubt, try to explain your confusion as part of the answer and maybe you will receive partial credit.

[1] The pledge is "I pledge my honor that I have not violated the honor code during this exam and followed all instructions."

1. (20 points) Consider the set of well-formed regular expressions as a language over the alphabet $\{\cup, \cap, \circ, *, 0, 1\}$. Show that it is not regular.

2. (25 points) Show that the following language is undecidable. (The constants 100 and 1000 are not too important so don't get hung up on them.)

$$\{< M >: M \text{ is a TM that runs in } 100n^2 + 1000 \text{ time}\}.$$

3. (40 points) For each of the following statements, indicate if they are `True`, `False`, or `Open`. Give a brief reason. Do not assume any unproven hypotheses. (+5 points for a correct answer, -2 for an incorrect answer, and 0 points if left blank.)

   (a) Every regular language is in L.

   (b) $\{0^n1^n : n \geq 0\}$ is NP-complete.

   (c) 3SAT is PSPACE-complete.

   (d) TQBF is NP-hard.

(e) TQBF is in NL.

(f) 3SAT is in BPP.

(g) If P = NP then one-way functions do not exist.

(h) A_{TM}, the language corresponding to the halting problem, has interactive proofs.

4. (20 points) Suppose we are given two programs P_1, P_2 and told that one of them decides 3SAT in $O(n^3)$ time but aren't told which one. Describe how to use them as

subroutines to write a polynomial-time algorithm that solves 3SAT.

5. (25 points) In the general version of the traveling salesman problem the input is a set of nodes and for each pair i, j of nodes a distance $d_{i,j} \geq 0$ between i and j. These distances can be arbitrary and need not satisfy triangle inequality. The goal is to find the shortest cycle that visits each node exactly 1. Show that for every constant $c > 1$ if there is a polynomial-time approximation algorithm for this problem that

approximates it within a factor c then P = NP. (Hint: Reduce from UHAMPATH.)

THAT'S ALL FOLKS; NEXT TWO PROBLEMS ARE OPTIONAL

6. (25 points) Assume that the usual axiomatic system for number theory is consistent. Describe a Turing machine M such that the statement "M runs in $100n^2 + 1000$ time" is unproveable in this axiomatic system.

7. (20 points) Suppose there is a Turing machine R that runs in $O(n^3)$ time and it can decide the satisfiability of every boolean formula whose encoding length is of the form 2^k for some k. We have no guarantee on the performance of R on other boolean formulae.

   (a) (10 points) If such a machine exists, can we conclude P = NP? Explain your answer.

   (b) (10 points) Suppose the machine R only decided satisfiability for formulae of length 2^{2^k} where k is an integer. Now can we conclude P = NP?