The Spite Motive and Equilibrium Behavior in Auctions*

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Abstract

We study auctions where bidders have independent private values but attach a disutility to the surplus of rivals, and derive symmetric equilibria for first-price, second-price, English, and Dutch auctions. We find that equilibrium bidding is more aggressive than standard predictions. Indeed, in second-price auctions it is optimal to bid above one's valuation; that is, bidding "frenzies" can arise in equilibrium. Further, revenue equivalence between second-price and first-price auctions breaks down, with second-price outperforming first-price. We also find that strategic equivalence between second-price and English auctions no longer holds, although they remain revenue equivalent. We conclude that spiteful bidding rationalizes anomalies observed in laboratory experiments across the four auction forms better than the leading alternatives.

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1 Introduction

1.1 Background

Frenzied bidding in auctions is popularly ascribed to irrational behavior. Bidders are pictured as getting carried away in their bidding and paying prices that exceed their value for an object. When bidders know their valuations for an object, it would seem that paying a price above one's value is possible only in the face of such irrational behavior. The main goal of this paper is to put such "frenzied" behavior on a rational footing, much in the spirit of Bulow and Klemperer (1994), who study apparently irrational price movements in a sequence of sales in an asset market.

Our model departs from the standard framework in that we postulate that a bidder cares not only about her own surplus in the event that she wins the auction, but also

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about the surplus of her rivals in the event that she loses the auction. Such motives are frequently referred to in anecdotal accounts of auctions, especially for highly prized and rare collectibles. The nineteenth century art expert Edmond Bonnaffé is reported to have observed that[1] "...after Michelangelo's pictures and the Medici porcelain the rarest thing he had ever seen among collectors was goodwill, and he drew the conclusion that collectors' mania embraced the desire to own things for oneself, the desire to own them for others, and the desire to stop other people owning anything."

On a more mundane level, with commodity items rather than prized *objets d'art*, bidders might care about a rival's surplus for sound commercial reasons—a rival's surplus might affect a bidder's competitive position at the conclusion of the auction. In his classic survey of real auctions around the world (ca. 1960), Cassady (1967, p. 145) recommends that "...a buyer should concentrate his attention on items of interest to him, and not become embroiled in the emotional atmosphere of an auction. He may, however, bid on a lot in order to force a competitor to pay a higher price."

Regardless of whether the spiteful motive derives from envy or a shrewd calculation, we show that when such considerations are present, rational bidding can lead to a "frenzy" even when bidders have independent private valuations for the object being auctioned. Indeed, in second-price auctions (and English auctions when only two active bidders remain), we show that optimal bidding (almost) always entails bidding *above* one's valuation. Further, our model predicts that the form of the auction affects the degree of frenzied bidding. We show that, when bidders care about the surplus of their rivals, English (and second-price sealed bid) auctions outperform first-price auctions in terms of expected revenues to the seller.

1.2 The experimental evidence

While auction theory is generally viewed as one of the most successful real-world applications of information economics, a number of puzzling discrepancies between the predictions of theory and behavior in laboratory experiments have been observed (see Kagel, 1995 and Davis and Holt, 1993 for useful surveys of auction theory and laboratory experiments). In this paper we explore the consequences of a model for bidder behavior that incorporates, in addition to a utility for one's own surplus, a disutility for the surplus of a rival—interpretable as "spiteful" behavior. We show that equilibrium behavior under this model is at least as good, and in important cases, strictly better at explaining experimental results than the leading alternatives, with equal complexity—the addition of one real parameter (α) that represents the weighting given to the surplus of one's rivals.

At the risk of oversimplifying, three stylized facts have emerged from the experimental literature that represent anomalies between theoretically predicted and actual behavior:

Anomaly 1 Subjects consistently bid more aggressively than predicted by theory in sealed bid first-price auctions with independent private valuations. (See, for example, Holt and Sherman, 2000, who give explicit graphs of measured bid functions in this case.)

Anomaly 2 Subjects consistently bid more aggressively than predicted by theory in sealed bid second-price auctions. (See, for example, Kagel, Harstad, and Levin, 1987.)

[1] Rheims (1961, p. 7).

Anomaly 3 Strategic equivalence between English auctions and second-price sealed bid auctions fails, with far less overbidding in English auctions than in second-price sealed bid auctions. (Again, see Kagel, Harstad, and Levin, 1987 for a good example of clear experimental evidence of this phenomenon.)

A variety of explanations have been offered to account for these departures from theory. In the case of first-price auctions, the leading explanation is risk aversion among subjects. Riley and Samuelson (1981) showed theoretically that risk aversion leads to more aggressive bidding in this auction, and attempts to fit parametric classes of risk averse preferences to data on these auctions have proved useful. While explanations emphasizing risk aversion work well in explaining Anomaly 1, they fail to explain Anomalies 2 and 3, since behavioral predictions in both second-price and English auction forms derive from dominance, and hence are independent of risk preferences.

An alternative explanation advanced for differences between second-price and English auctions emphasizes the difference in feedback subjects receive when they overbid in these two auction forms (see Kagel, Harstad, and Levin). Whereas the drawbacks associated with overbidding seem to be immediately apparent to subjects participating in English auctions once prices rise above a bidder's valuation, in sealed bid settings, a subject receives negative feedback from overbidding only in circumstances where the second highest bid lies between her[2] bid and her valuation. Thus, subjects are more apt to be able to "get away" with overbidding in this auction without recognizing the potentially adverse consequences of this strategy. This point tends to explain Anomaly 3 (although only qualitatively), but has little to say about why subjects overbid in the first place.

Having just neatly characterized the experimental literature, we should point out that it is complex, and sometimes ambiguous or incomplete.[3] Thus, while we argue that our model is effective in rationalizing several experimental results, we will also derive predictions for experiments that remain to be done, and which would be able to validate or falsify our model, as well as differentiate it from competing models of behavior (these might be termed "out of sample predictions"). We view this feature of the model as being no less important than its power to explain current facts as they are understood.

1.3 Summary of results

We conclude the introduction by summarizing our theoretical results. Throughout the paper we restrict attention to single-object, independent private value (IPV) auctions, with valuations identically distributed among bidders and consider the four most important auction forms: first-price sealed bid, Dutch, second-price sealed bid (Vickrey), and English absent reserve prices. Our focus on this class of auction problems is driven by two considerations: First, the impact of spiteful preferences on bidding is most transparent in this standard setting. Adding complication does little to affect the underlying economics of spiteful bidding. Second, many of the laboratory experiments on auctions use exactly

[2]Throughout, odd numbered bidders are females, even males.

[3]Rarely is the same experiment repeated with identical protocols and experimental conditions. For example, as pointed out by Kagel in his survey, at least one study of behavior in second-price auctions (Cox, Roberson, and Smith, 1982) explicitly prohibited bidding above valuations, thereby legislating away an effect predicted by our model. As another example, Kagel cites the reported failures of strategic equivalence in second-price and English auctions for experiments with affiliated values (Kagel, Harstad and Levin), but cites these experimental results in the unaffiliated values case, using the fact that—theoretically—dominant strategy is not affected by affiliation.

this setting, making it the most relevant of one seeks to rationalize the three anomalies described above.

Our main findings are then:

(a) For any positive weight given to the payoffs of a rival bidder, equilibria in the first-price and second-price auctions entail overbidding compared to the case where spite motives are absent. Indeed, when spiteful incentives matter, it is no longer the case that bidding one's value is a dominant strategy in the second-price auction.

(b) Compared to the second-price sealed bid auction, equilibrium bidding in the English auction tends to be less aggressive until only two bidders remain. Thus, strategic equivalence between these two auction forms fails when spiteful incentives are present.

(c) When bidders behave spitefully, these auction forms are no longer revenue equivalent. Instead, second-price auctions outperform first-price auctions.

The remainder of the paper proceeds as follows: In Section 2, we describe the model for spiteful behavior precisely, and discuss support for it from fields outside auction theory. In Section 3 we derive symmetric Nash equilibria for the four canonical auction forms—First-price, second-price, Dutch, and English—and highlight the differences between our predictions and those of the standard theory. In Section 4, we examine testable predictions of our model. Section 5 concludes.

2 Model

Suppose that n risk-neutral bidders numbered $i = 1, ..., n$ compete in a single object auction. If bidder i receives the object, she values it at v_i where v_i is drawn from some smooth and atomless distribution F with finite support. Without loss of generality, we normalize the support to be $[0, 1]$. The density associated with F is f. In addition to deriving surplus when she receives the object, bidder i is concerned with (envious of) the surplus of rival bidders in the event she does not receive the object. This concern, which we interpret as a spiteful motive, is reflected by a term in bidder i's utility function, with negative weight α. Note that this utility function will be used to determine i's equilibrium bidding function, but will not affect actual monetary payoffs, which are conventional. Even though bidder i takes rivals' surplus into account in bidding, when bidder i loses an auction, she neither pays nor receives money.

In allocating the object, bidder i submits a bid b_i. Let $b = (b_1, b_2, ..., b_n)$ denote the vector of bids submitted in the auction. The auction rules prescribe an allocation rule and a payment rule as a function of the vector of bids. Let $p_i(b)$ denote the payment required of bidder i when the bid vector b is submitted. In the auction forms we consider, the payment rule is such that no payment is required of i unless the object is allocated to i. When i is allocated the object, we shall say that i "wins" the auction. Thus, bidder i's utility is:

$$\Delta = \begin{cases} v_i - p_i(b) & \text{if } i \text{ wins} \\ -\alpha(v_k - p_k(b)) & \text{if } k \neq i \text{ wins} \end{cases}$$

The notation Δ is a mnemonic for the fact that, in our model, a bidder is concerned with the weighted difference between her own surplus and that of her rivals. Thus, we can write bidder i's optimization problem as choosing b_i to maximize the expectation of Δ. Notice that the standard IPV case arises when $\alpha = 0$.

In a dynamic setting, such as the English auction, the number of rivals can change. Since we will adopt the same model for English auctions as Milgrom and Weber (1982), the so-called "button" or "Japanese" auction, in which the bidders know at any time how many rivals remain active, this can, in general, matter. It will turn out, however, that when we analyze bidding in the English auction, the equilibrium strategy we consider is independent of the number of rivals—just as it is in the case when spite is not present and truthful bidding is a dominant strategy.

Example

To illustrate how standard auction theory results are affected by introducing spite motives, consider a very simple case, a Vickrey auction (Vickrey, 1961) where two bidders are competing and valuations are drawn from the uniform distribution. Suppose equal weight is given to the rival's surplus; that is, $\alpha = 1$. If bidder 2 bids sincerely (bids his valuation), then bidder 1's optimization problem becomes choosing b_1 to maximize the expectation of

$$\Delta(v_1, v_2) = [v_1 - v_2] \cdot I_{b_1 \geq v_2} - [v_2 - b_1] \cdot I_{b_1 < v_2}, \tag{1}$$

where I is the indicator function; or, equivalently,

$$\begin{aligned} \mathrm{E}_{v_2}(\Delta(v_1, v_2)) &= \int_0^{b_1} (v_1 - v_2) dv_2 - \int_{b_1}^{1} (v_2 - b_1) dv_2 \\ &= b_1 v_1 - 1/2 + b_1 - b_1^2. \end{aligned} \tag{2}$$

The solution to this optimization is to bid $b_1 = (v_1 + 1)/2$. That is, bidding *above* one's value is (almost) always optimal.

When bidder 1 follows this strategy, her expected payoff is $\mathrm{E}(\Delta) = 1/12$ compared with an expected payoff of zero under sincere bidding. The point is that by bidding above her value, bidder 1 incurs losses in the event that 2 bids between 1's value and her bid; however, this is more than offset by forcing bidder 2 to pay a higher price for the object when 2 wins the auction.

Notice also that the degree of overbidding, the difference between the optimal bid and a sincere bid, gets smaller as 1's valuation gets higher. The intuition here is that as bidder 1's bid gets higher, it grows increasingly less probable that 1's bid will affect 2's price and this attenuates the incentive to overbid. At the upper support of the distribution, there is no chance that 1's bid will affect 2's price, so 1 bids sincerely.

Robustness of the model

As with all mathematical models, and particularly those that attempt to model economic behavior, it is important to scrutinize the sensitivity of the results to the particular choices made. In this section we will give a heuristic argument that shows that the overbidding that occurs in our model is a general property of any model that takes into account a bidder's envy for the surplus of rivals.

Suppose we consider the maximization of a general, smooth utility function $U(s_1, s_2)$, where s_1 and s_2 are the expected surpluses of bidders 1 and 2, and for simplicity we restrict attention to the two-bidder case. We think of the valuation v_1 as given, and we choose the bidding function b in equilibrium so that U is stationary when both bidders adopt it. Now the total derivative of the utility is

$$\frac{dU}{db} = \frac{\partial U}{\partial s_1}\frac{\partial s_1}{\partial b} + \frac{\partial U}{\partial s_2}\frac{\partial s_2}{\partial b} \ . \tag{3}$$

Note that in considering the effects of perturbing b, we fix the bidding strategy of bidder 2 at $\beta = b$. In the standard formulation, when U does not depend on s_2, U is maximized by the equilibrium choice of b, and $\partial U/\partial s_1 = 0$. We start with this situation and consider the effect of introducing a new dependence of U on s_2.

From Eq. 3 we can see that the effect of making U depend on s_2, the rival's surplus, is to shift the solution for b. The first term by itself has a downward zero crossing at the original equilibrium. If the second term, $(\partial U/\partial s_2)(\partial s_2/\partial b)$ is positive, the zero crossing shifts to the right (overbidding), and vice versa. Now, we can assume that in modeling spiteful behavior that $\partial U/\partial s_2$ is negative, since we want to represent the situation when the surplus of bidder 1's rival decreases 1's utility. The remaining factor, $\partial s_2/\partial b$, is then critical. In the first-price case,

$$s_2 = \int_{\beta^{-1}(b)}^{1} [v_2 - \beta(v_2)] f(v_2) dv_2 \ , \tag{4}$$

and

$$\frac{\partial s_2}{\partial b} = -\frac{1}{b'}(v_1 - b(v_1)) < 0 \ , \tag{5}$$

where we use the fact that there is no overbidding in the standard case (our starting point).

In the second-price case, now denoting the bidding function by γ,

$$s_2 = \int_{\gamma^{-1}(b)}^{1} [v_2 - b] f(v_2) dv_2 \tag{6}$$

and, similarly,

$$\frac{\partial s_2}{\partial b} = -\frac{1}{b'}(v_1 - b(v_1)) - (1 - F(v_1)) < 0 \ . \tag{7}$$

The same argument applies to the final (two-bidder) stage of the English auction.

We conclude from this argument that, at least in the first-price, second-price and English auctions, the initial perturbation effect of one bidder envying the surplus of another, for quite general utility functions, is to move bids higher. This confirms our specific results, and shows that they are in this sense robust with respect to the particular way that the utility function reflects rivals' surpluses.

It is also easy to work out the following variation. If the utility function is altered so that what matters to a bidder is not the surplus of the winner, but rather the surplus of an arbitrary representative from among her rivals, both the overbidding and the revenue ranking for first- and second-price auctions are preserved. This provides further evidence for the robustness of the qualitative results with respect to variations in the particular utility function used.

Related literature

There are a variety of economic settings where agents seem to care about the surplus of their rivals. For instance, there have been significant concerns in spectrum auctions that, given liquidity constraints, a firm could gain a competitive advantage over its rivals if the rivals were forced to overpay to acquire certain licenses. In corporate mergers the price paid by an acquiring company to obtain some target company will have an effect on competitiveness *ex post*; thus potential acquiring companies would certainly care about this relative profit effect in determining their bids. Viewed in this light, our work is somewhat related to Jehiel, Moldovanu and Stacchetti (1996), who study auctions where bidders impose externalities on one another. Their main concern is with finding revenue maximizing schemes, and this typically entails much more complicated mechanisms than those we study. Closer in spirit to our analyis is Jehiel and Moldovanu (2000). Here, the effect of post-auction competition on bidding in an IPV setting is studied. In contrast with our paper, their main concern is with analyzing optimal reserve prices and entry fees in second-price sealed bid auctions where there is subsequent competition among competing bidders post auction. Also somewhat related is Koçkesen, Ok, and Sethi (2000), who consider the effect of incorporating relative profit motives in a general game-theoretic setting, and describe applications to the delegation of decision making in oligopolistic industries, and to preference evolution.

More broadly, a number of researchers have studied auctions in which a bidder's surplus is connected to those of other bidders or to the seller's revenue (see, for example, Engelbrecht-Wiggans, 1994; Bulow, Huang, and Klemperer, 1999; Goeree and Turner, 2000; and Engers and McManus, 2001). Examples include auctions to raise money for a public good, corporate takeover battles when combatants have toeholds, and buyer-ring knockout auctions. While the models in these cases are somewhat different from ours, with quite different motivations, the equilibrium bidding functions in first and second-price auctions are of the same form, and, as we shall see, previous work on ranking the revenue in these two auction forms can be appropriated for our purposes.

Our paper is also related to papers by Ettinger (2002) and Das Varma (2002), who both study auctions where a winning bidder may exert an externality on a losing bidder. Ettinger studies the case where two firms are bidding for a single object and each firm has a symmetric financial stake in the other firm's profits. Thus, a firm which loses the auction derives a *benefit* proportional to the profits (surplus) of its rival. In contrast, we study the case of n bidders, where all losing bidders suffer a *cost* in utility proportional to the surplus of the winning bidder. Nonetheless, equilibrium bidding in the first-price auction in Ettinger is identical but for the sign of the externality term (for $n = 2$). This reflects the difference in the direction of the externality between the two papers. In the second-price auction, the differential equation implicitly defining equilibrium bidding is likewise identical to ours but for the sign of the externality term. However, in the second-price auction, the difference in the direction of the externality also affects the *boundary condition*. In Ettinger's paper, a bidder with the lowest type has no incentive to bid above its value since its price determines the surplus of the winning bidder and smaller surpluses to the winner mean smaller payoffs to the loser. In contrast, a bidder of the lowest type in our paper has a positive incentive to bid above his value since reducing the winner's surplus is a benefit to the losing bidder. Instead, the boundary condition in our analysis is determined by the bidder with the highest type, who has no incentive to bid above his value for the usual Vickrey reasons. This difference in the boundary conditions accounts for the fact that the derived equilibrium bidding strategies in the two papers are different for the second-price and English auctions. Another important difference is that, as pointed out earlier, the utility in our model determines equilibrium behavior, but not monetary payoffs.

Das Varma analyzes a situation where a winning bidder enjoys surplus in the usual way but a losing bidder may suffer a loss of a *fixed amount* depending on the identity of the winning bidder. Note, however, that since the loss is a fixed amount rather than depending on the surplus of the winning bidder, the analysis in that paper differs considerably from ours.

Relative to this literature, ours is the first paper to suggest spiteful bidding as a motive for rationalizing anomalous auction results in laboratory experiments. We are also the first to point out the observational equivalence in first-price auctions between bidding behavior with externalities and bidding behavior under risk aversion. Finally, we are the first to offer testable comparative static predictions for distinguishing the spite motive from alternative behavioral hypotheses in laboratory experiments.

Psychological motives in experiments

Outside of the auction context, a number of theories have emphasized the importance of spite motives in affecting the behavior of subjects in experiments. Perhaps the most prominent example is the rejection of unequal offers in ultimatum games. The spite motive plays a key role in the analysis of Levine (1998), who shows that incorporating spite and altruism in subject preferences explains apparent deviations from equilibrium behavior in a variety of games. The spite motive also has a prominent role in Rabin's (1993) seminal article on incorporating fairness in game theory using psychological games. Saijo and Nakamura (1995) interpret their results in a public goods game as evidence of spiteful behavior. They characterize their subjects as having "... first priority not the total amount of payoff they receive but the ranking among them."

Spite is not the only or even the most prominent motive in describing subject behavior in certain experiments. Fehr and Schmidt (1999) and Bolton and Ockenfels (1999) both stress inequality aversion as an underlying explanation of the taste for "fairness" exhibited by subjects in many games. In some public goods games and prisoner's dilemmas, subjects seem to exhibit much more altruism than standard theory predicts. In contrast, such altruistic behavior seems absent in experiments in double auctions. Thus, while spite is unlikely to be the only psychological factor motivating behavior in auctions, we think it appears sufficiently prominently to consider it as an alternative to others, especially risk aversion, which is widely used to explain overbidding in first-price auctions. We do so in this paper by adding one degree of freedom (α) to the standard profit-maximizing model, so our spite model and the usual risk averse model (constant relative risk aversion parameter ρ) are equally parsimonious.

A variety of natural settings, such as competition for food or mates, are in the nature of auctions (see for instance Bishop, Canning, and Maynard Smith, 1978; and Maynard Smith, 1982), where the objective of a competitor is simply to outperform his or her rivals. Our model is exactly along these lines. In fact, the consideration of spiteful and altruistic behavior has been important in evolutionary biology for some time. Hamilton (1971) provides an early example of the consideration of spite as a determinant of biological fitness in an evolutionary context. He explains his use of the term as follows: "Let us call an action which harms others without benefiting the self 'spiteful,' and if a spiteful action involves harm to the self, let us call it 'strongly spiteful.'" Thus, bidders in our model with a positive α are in Hamilton's sense strongly spiteful.

The recent subculture of internet auctions provides anecdotal evidence for the spite motive. The common term "revenge bidding" is used in exactly this way: bidding with the intent to raise the price that a rival bidder, perceived as willing to go higher, must pay. While some laboratory experiments have provided evidence that subjects can behave altruistically (corresponding to $\alpha < 0$ in our model), it hardly seems plausible to attribute altruism to bidders in real auctions, which are so explicitly and notoriously competitive.

3 Analysis

In this section we derive a symmetric equilibrium where bidders choose differentiable strictly increasing bidding strategies in auctions where spite motives are present for the four canonical auction forms.[4] We begin (in Subsection 3.1) by constructing bidding equilibria for the case where $n = 2$.[5] The case is useful because it illustrates, in a transparent fashion, how spiteful incentives affect equilibrium bidding. We turn to the general case in Subsection 3.2 and construct symmetric equilibrium bidding strategies for the first-price and Dutch auctions (Proposition 1), the second-price auction (Proposition 2), and the English auction (Proposition 3).

3.1 Two-bidder auctions

First, consider the case where two spiteful bidders compete in a first-price auction. The payoff function in this case as a function of values v_1, v_2 is

$$\Delta(v_1, v_2) = [v_1 - b] \cdot I_{b \geq \beta(v_2)} - \alpha[v_2 - \beta(v_2)] \cdot I_{b < \beta(v_2)}. \tag{8}$$

where bidder 1 believes that bidder 2 is employing the bidding function $\beta(v_2)$ and I is the indicator function. Assuming that bidder 2's bidding strategy is strictly increasing, bidder 1's objective is to choose a bid b to maximize the expectation over v_2 of her payoff,

$$\mathrm{E}_{v_2}(\Delta) = \int_0^{\beta^{-1}(b)} [v_1 - b] f(v_2) dv_2 - \alpha \int_{\beta^{-1}(b)}^{1} [v_2 - \beta(v_2)] f(v_2) dv_2. \tag{9}$$

Let $b(v)$ be the strategy implied by this optimization. At a symmetric equilibrium $\beta(v) = b(v)$. Differentiating Eq. (9) with respect to b, setting the result to zero and letting $\beta(v) = b(v)$ then yields, after rearrangement, the following linear differential equation for $b(v)$:

$$b'(v) + \left[\frac{(1+\alpha)f}{F}\right] b(v) = \left[\frac{(1+\alpha)f}{F}\right] v. \tag{10}$$

Multiplying by the integrating factor[6] $F(v)^{1+\alpha}$ yields the solution

$$b(v) = v - \frac{\int_0^v F(t)^{1+\alpha} dt}{F(v)^{1+\alpha}}, \tag{11}$$

where we use the condition that $b(0)$ is finite to obtain the constant of integration. We may rewrite Eq. (11) as:

$$b(v) = \mathrm{E}(y | y \leq v) + \left[\frac{\int_0^v F(t) dt}{F(v)} - \frac{\int_0^v F(t)^{1+\alpha} dt}{F(v)^{1+\alpha}}\right]. \tag{12}$$

The first part of this expression represents the bidding strategy in the standard case where spite motives are absent. The term in square brackets is the impact of these preferences

[4] Our focus on symmetric equilibria is standard for analyzing behavior in laboratory settings where bidders are typically anonymously matched.

[5] The construction here is heuristic since formally showing that these strategies comprise a symmetric equilibrium follows from Propositions 1 and 2.

[6] The integrating factor for a linear differential equation of the form $b'(v) + P(v)b(v) = Q(v)$ is $e^{\int P(v)dv}$. See, for example, Kells (1954).

on equilibrium bidding. Clearly, when $\alpha = 0$ this term is zero. For all $\alpha > 0$, the term in the square brackets is positive. Thus, first-price auctions where spite motives are present give rise to more aggressive bidding than those in which these incentives are absent. As we show in Proposition 1, this effect on bidding generalizes to $n > 2$ bidder settings. To obtain some intuition for this effect, consider the case where both bidders are following the standard equilibrium strategy $b(v) = \mathrm{E}(y|y \leq v)$. In this case, the marginal benefit from bidding slightly higher and thereby increasing the chances of winning is exactly offset by the marginal cost of paying a higher price for the object conditional on winning. In an auction where bidders have spiteful incentives, an additional marginal benefit term is present; bidding a bit higher reduces one's rival's chance of winning and hence his expected surplus. Thus, it pays to raise one's bid in these circumstances. Equilibrium occurs where this additional marginal benefit term is compensated by a higher marginal cost of raising one's bid, which occurs with a more steeply sloped bidding strategy.

This may be readily seen for the case where values are uniformly distributed. In this case, $b(v) = [(1+\alpha)/(2+\alpha)]v$. Notice that the greater are the spite motives (that is, α larger), the more steeply sloped is the equilibrium bidding strategy.

Next, we turn to the second-price (or Vickrey) auction. The payoff to bidder 1 is the same as in the first-price auction (Eq. (8)) except that the positions of the terms $\beta(v_2)$ and b are interchanged.

$$\Delta(v_1, v_2) = [v_1 - \beta(v_2)] \cdot I_{b \geq \beta(v_2)} - \alpha[v_2 - b] \cdot I_{b < \beta(v_2)}. \tag{13}$$

The expectation over values of v_2 assuming $\beta(v_2)$ is a strictly increasing strategy becomes

$$\mathrm{E}_{v_2}(\Delta) = \int_0^{\beta^{-1}(b)} [v_1 - \beta(v_2)] f(v_2) dv_2 - \alpha \int_{\beta^{-1}(b)}^1 [v_2 - b] f(v_2) dv_2, \tag{14}$$

and the resulting equilibrium condition for $b(v)$ now becomes

$$b'(v) + \left[\frac{((1+\alpha)/\alpha)f}{(F-1)}\right] b(v) = \left[\frac{((1+\alpha)/\alpha)f}{F-1}\right] v. \tag{15}$$

The integrating factor $(F-1)^{(1+\alpha)/\alpha}$ then yields the solution

$$b(v) = v + \frac{\int_v^1 (1 - F(t))^{(1+\alpha)/\alpha} dt}{(1 - F(v))^{(1+\alpha)/\alpha}}, \tag{16}$$

where we use the condition that $b(1)$ is finite to find the constant of integration. As is apparent from Eq. (16), equilibrium bidding in second-price auctions where spite motives are present entails bidding above one's valuation. To obtain some intuition for overbidding in these auctions, it is again useful to consider the case where both bidders are following the standard equilibrium strategy, which now means bidding one's value. When spiteful incentives are absent, raising one's bid leads to a marginal gain from the increase in probability of winning, but this is exactly offset by the marginal cost of winning at a price in excess of one's valuation. Spiteful incentives introduce two new sources of marginal benefit. First, increasing one's bid denies a chance at one's rival's winning the object. Of course, in the case where bidders are bidding sincerely, this benefit term is exactly zero, since only rivals with zero surplus are losing a chance at winning. In addition, by raising one's bid, one increases the price of the rival bidder in the event he has a higher valuation. This term, $\alpha(1 - F(v_i))$, is first-order, so it pays to bid above one's valuation.

In equilibrium, bidding above valuation raises the marginal cost term to just compensate for these two new marginal benefit terms.

Notice, however, that the expression $\alpha(1-F(v_i))$ is zero when a bidder has the highest possible valuation. In this case, incentives are identical to the standard case and bidding sincerely is optimal. Thus, unlike the situation in a first-price auction, where the amount of overbidding grew as a bidder's valuation increased, in the second-price auction just the opposite is the case. The incentives to overbid are the greater for bidders with relatively low valuations and overbidding is a *decreasing* function of one's valuation.

This is readily illustrated for the case where valuations are drawn from the uniform distribution. Here, the equilibrium bidding strategy takes the simple form

$$b(v) = \left(\frac{1+\alpha}{1+2\alpha}\right) v + \frac{\alpha}{1+2\alpha}. \tag{17}$$

Notice that $b(1) = 1$, while $b(0) = \alpha/(1+2\alpha)$. Moreover, the rate by which bids increase in valuations is $(1+\alpha)/(1+2\alpha) < 1$. That is, when spite motives are present, bids go up more slowly as a function of valuation than in the standard case.

Dynamic auctions

We now turn to the dynamic analogs of the first and second-price auctions. For the usual reasons, the Dutch auction remains strategically equivalent to the first-price auction, so we focus attention on the English auction. In contrast to the standard case, where bidding sincerely is a weakly dominant strategy, we have already shown that bidding sincerely in a second-price auction is no longer optimal. We shall show that neither is it the case in the English auction.

We model English auctions as button auctions, exactly as in Milgrom and Weber (1982). We think of each bidder as keeping her finger on her button, and releasing it (irreversibly) when she is no longer willing to pay the ascending price. We say she is "active" when her finger is on the button, and that she "drops out" when she releases her button. All bidders know at all times the number of currently active bidders. Throughout, we refer to the price at which a bidder releases his or her button as the bidder's "drop out strategy."

Suppose that both bidders are active when the clock shows a price of a. At that point, bidder 1 knows her value v_1. Further, she can infer that if bidder 2 is following a strictly increasing drop out strategy, then his valuation is above some threshold, a, and has density

$$g(v) = \frac{f(v)}{1-F(a)}, \tag{18}$$

and the distribution

$$G(v) = \frac{F(v)-F(a)}{1-F(a)}. \tag{19}$$

Suppose, in fact, that from this point onward, bidder 1 conjectures that 2 is employing the strictly increasing drop out strategy $\beta(v)$. Then the payoff of bidder 1 may be written as

$$\mathrm{E}_{v_2}(\Delta) = \int_a^{\beta^{-1}(b)} [v_1 - \beta(v_2)] g(v_2) dv_2 - \alpha \int_{\beta^{-1}(b)}^1 [v_2 - b] g(v_2) dv_2. \tag{20}$$

In a symmetric equilibrium, this yields the same solution as in the second-price sealed bid auction, save for the fact that F is replaced by G. Thus

$$b(v) = v + \frac{\int_v^1 (1-G(t))^{(1+\alpha)/\alpha} dt}{(1-G(v))^{(1+\alpha)/\alpha}}. \tag{21}$$

However, substituting the definition of G into this equation leads back to Eq. (16). Thus, a symmetric equilibrium in the two bidder English auction is to drop out at a price equal to one's bid in a second-price sealed bid auction. Moreover, if the clock already exceeds this level, it is optimal to drop out immediately. For future reference, we summarize these observations as Lemma 1.

Lemma 1: *Suppose two bidders remain in an English auction. Then a symmetric equilibrium is to drop out at a price equal to one's bid in a two bidder second-price sealed bid auction. If the current price already exceeds this level, then it is optimal to drop out immediately.*

Thus, in the two bidder case, strategic equivalence continues to hold between second-price and English auctions. However, as we shall show in the next subsection, this property does not hold more generally. Nonetheless, bidding in the two bidder subgame will be crucial in constructing equilibrium bidding strategies in an n-bidder English auction.

3.2 n-bidder auctions

First-price and Dutch auctions, n bidders

As usual, we conjecture that all rivals are playing the strictly increasing and differentiable bidding strategy $\beta(v)$, and we look for a symmetric equilibrium. Considering a first-price auction with n spiteful bidders, we can write 1's profit as

$$\Delta = [v_1 - b] \cdot I_{\beta(v_2)\leq b} \cdots I_{\beta(v_n)\leq b} - \alpha(n-1)\,[v_2 - \beta(v_2)] \cdot I_{\beta(v_2)\geq b} \cdot I_{v_3\leq v_2} \cdots I_{v_n\leq v_2}. \quad (22)$$

Here we have arbitrarily chosen to write the surplus of rival 2, and weighted his profit by $(n-1)$, since the $(n-1)$ rivals are indistinguishable and have identical surplus distributions. This yields the necessary condition

$$b'(v) + \left[\frac{(n-1)(1+\alpha)f}{F}\right] b(v) = \left[\frac{(n-1)(1+\alpha)f}{F}\right] v. \quad (23)$$

This is just Eq. (10) with $1+\alpha$ replaced by $(n-1)(1+\alpha)$, and the solution is of the same form. Of course, this is merely necessary. In Section A.1 we establish sufficiency. Thus, we have shown

Proposition 1: *A symmetric equilibrium for the first-price auction, as well as the Dutch auction, is*

$$b(v) = v - \frac{\int_0^v F(t)^{(n-1)(1+\alpha)}dt}{F(v)^{(n-1)(1+\alpha)}}. \quad (24)$$

We can rewrite Eq. (24) as

$$b(v) = E\left[y|y\leq v\right] + \left[\frac{\int_0^v F(t)^{n-1}dt}{F(v)^{n-1}} - \frac{\int_0^v F(t)^{(n-1)(1+\alpha)}dt}{F(v)^{(n-1)(1+\alpha)}}\right]. \quad (25)$$

and once again it is clear that bidding is more aggressive in first-price auctions when spite motives are present than when bidders do not care about their relative position.

When values are distributed uniformly, the equilibrium bidding strategy is

$$b(v) = \left(1 - \frac{1}{n+\alpha(n-1)}\right) v. \quad (26)$$

Second-price auctions, n bidders

The derivation of the necessary condition in the second-price case proceeds along the same lines as in the first-price case, the only extra complication being that the cost to 2 when 2 wins must be broken down into two subevents: the event that 2 wins and 1 is second highest (in which case 1's bid determines what 2 pays), and the event that 2 wins and one of $3, ..., n$ is second-highest (in which case that bidder determines what 2 pays). The resulting differential equation and necessary condition turns out to be identical to that for the $n = 2$ case, Eq. (15), and we repeat the solution here for reference. We establish sufficiency in Section A.1.

Proposition 2: *A symmetric equilibrium for the second-price auction is:*

$$b(v) = v + \frac{\int_v^1 (1 - F(t))^{(1+\alpha)/\alpha} dt}{(1 - F(v))^{(1+\alpha)/\alpha}}. \tag{27}$$

Notice that once again this is in the form of the absolute-profit bid (in this case truthful) plus an overbidding term that is always positive. Thus, one sees that bidding *above one's value* is indeed optimal whenever weight is placed on the loss of rival bidders.

When values are distributed uniformly, the equilibrium bidding strategy for n bidders is of course the same as before,

$$b(v) = \left(\frac{1+\alpha}{1+2\alpha}\right) v + \frac{\alpha}{1+2\alpha}, \tag{28}$$

regardless of n.

English auction, n bidders

We now turn to English auctions. Unlike the situation in a sealed bid auction it is now possible for a bidder to condition her strategy on the number of remaining active bidders in the auction as well as the current price. In the two bidder case, this did not matter—English and second-price auctions were strategically equivalent. We now show that this is not true in general.

We start with the last stage of the auction and work backwards. When only two active bidders remain, we showed (Lemma 1) that they could do no better than to use a drop out threshold equal to their bid in a two person sealed second-price auction. When more than two bidders remain active, a bidder contemplating dropping out at the current price knows that her "bid" will not affect the final price paid; thus, the usual incentive for overbidding in auctions where spite motives are present is effectively absent for such a bidder. We show in Proposition 3 below that such a bidder can do no better than to use a sincere bidding strategy in deciding when to drop out.

Proposition 3: *A symmetric equilibrium for the English auction is:*[7]

- *If three or more bidders are active, bidders drop out when clock price = value.*

[7] While we have constructed an equilibrium where all but two bidders bid truthfully, in fact there is a continuum of equilibria possible in the English auction. The construction for these is analogous to that given in Bikhchandani, Haile, and Riley (2001). Essentially, any construction whereby bidders initially drop out according to some increasing function of v (bounded from above by the drop out bids when only two bidders are active) constitutes a symmetric equilibrium of the English auction.

- *When only two bidders remain active, each drops out at the price she would submit in a second-price sealed bid auction. If the current price is already higher than this, each bidder drops out immediately.*

Proof: From Lemma 1, the strategies are clearly optimal when only two bidders remain active.

Suppose that at some price exactly three bidders are active with values v_1, v_2, and v_3. Without loss of generality, suppose that $v_1 > v_2$. We show that 3 can do no better than to follow the proposed equilibrium. For the rest of the proof we let $B(v)$ be the equilibrium bidding function for two bidders (actually not dependent on the number of bidders), Eq.(27).

Case 1: $v_3 < v_2$. Suppose that 3 drops out at a price at or below v_2 (this includes the equilibrium strategy). In this case, 3 earns $-\alpha \left(v_1 - B\left(v_2\right)\right)$. If, on the other hand, 3 stays in beyond price v_2, then, by lemma 1, 3 can do no better than to bid $B\left(v_3\right)$. Under this deviation, 3 earns $-\alpha \left(v_1 - B\left(v_3\right)\right)$, which is unprofitable since $B\left(v_3\right) < B\left(v_2\right)$. Thus, no profitable deviation is available to 3 in this case.

Case 2: $v_1 > v_3 > v_2$. If, when three bidders are active, 3 plans to drop out at a price at or above v_2 (this includes the equilibrium strategy), he earns $-\alpha \left(v_1 - B\left(v_3\right)\right)$. If, in contrast, 3 deviates by dropping out at a price below v_2, he earns $-\alpha \left(v_1 - B\left(v_2\right)\right)$. Since $B\left(v_2\right) < B\left(v_3\right)$, this is not a profitable deviation.

Case 3: $v_3 > v_1$. If 3 plans to drop out at a price at or above v_2 (this includes the equilibrium strategy), he earns at most $v_3 - B\left(v_1\right)$. In particular, following a drop out by 2 at price v_2, 3 could mimic the bid 2 would make when only two bidders remain active, in which case 3 earns $-\alpha \left(v_1 - B\left(v_2\right)\right)$. But, by lemma 1, we know that 3 prefers the bid $B\left(v_3\right)$ to any other feasible bid. Hence, $v_3 - B\left(v_1\right) \geq -\alpha \left(v_1 - B\left(v_2\right)\right)$. Now, consider a deviation where 3 drops out at a price below v_2. In this case, 3 earns $-\alpha \left(v_1 - B\left(v_2\right)\right)$, but this is not a profitable deviation. The cases where more than three bidders remain are analogous to where only three bidders are active. ■

Even when bidders have private valuations, since utility depends on interpersonal comparisons with other bidders, the information available in the English auction about the number of active remaining bidders strongly influences bidding—although for all but two bidders the influence is in the direction of sincere bidding rather than overbidding. The upshot is that there is more truthful bidding in an English auction than in a second-price sealed bid auction.

4 Testable implications

Up to this point, we have highlighted several ways in which our model leads to predictions that differ from the standard, risk-neutral model. In this section, we examine key testable implications of the equilibrium bidding strategies given in Propositions 1–3 that distinguish our model from several other alternative hypotheses. In Subsection 4.1, we study the performance of our model in fitting data in first-price auctions compared to the leading rival hypothesis—risk averse bidders. Our main result is to show, in Proposition 4, that the bidding model with spite motives fits first-price data exactly as well as the risk aversion hypothesis.

In Subsection 4.2, we suggest several ways to distinguish between the two theories, even in the case of first-price auctions. The main idea is that the spite motive operates through interpersonal comparison whereas risk aversion does not. Thus, our model predicts that aggressive bidding in first-price auctions should be greater when rivals are people than when rivals are machines. Risk aversion predicts no difference in bidding behavior under these two treatments. An alternative way to distinguish between bidding where incentives are spiteful and bidding under risk aversion, is through correlation of overbidding for a given subject in first and second-price auctions. Under our model, a subject who bids more aggressively in first-price auctions bids more aggressively in second-price auctions. Under risk aversion, an individual's deviations from risk-neutral bid predictions in first-price auctions should be uncorrelated with that same individual's deviations in a second-price auction.

In Subsection 4.3, we discuss the effects of varying the number of bidders. We point out that experiments with the English auction could distinguish between the spite and risk aversion models.

In Subsection 4.4, we derive a revenue ranking from equilibrium bidding strategies in our model. Our main result in this subsection is that second-price auctions outperform first-price auctions in the revenue metric. This revenue ranking is exactly the opposite of the prediction under risk aversion, thus providing yet another way to distinguish between the two models.

In Subsection 4.5, we contrast our model with a different alternative hypothesis—that bidders simply derive utility from the act of winning itself. Our model predicts that the divergence between equilibrium bidding strategies when bidders have spiteful incentives and equilibrium bidding in the standard model is (a) increasing in a bidder's value in a first-price auction and (b) decreasing in her value in a second-price auction. We show that the alternative hypothesis predicts divergence that is independent of a bidder's valuation for the object, thus providing a way to distinguish empirically between the two theories.

4.1 Risk aversion

The leading alternative theory to explain overbidding in first-price auctions is that bidders are risk averse, and this is reflected in their bidding. Incorporating risk aversion into preferences and then fitting the risk aversion parameter to the data has been shown to improve dramatically the fit between the theoretical predictions and laboratory outcomes in these auctions. A standard way to go about this (see Holt and Sherman, 2000) is to assume that bidders are symmetric and have preferences exhibiting constant relative risk aversion (CRRA) preferences, with risk aversion parameter ρ. That is, a bidder's preference is $u(t) = t^\rho$

How does the fit of these models compare to our predictions in explaining the data? The next proposition shows that the explanatory power of the two models is *exactly the same* in fitting first-price auction data. Specifically, we show in Section A.3, following Riley and Samuelson (1981), that the equilibrium bidding strategies when bidders have CRRA preferences are exactly the same as the equilibrium strategies given in our Proposition 1, provided that ρ and α are related by the one-to-one mapping $\rho = 1/(1+\alpha)$. We formalize this observation as:

Proposition 4: *Suppose that n risk averse individuals with preferences $u(t) = t^\rho$ compete in a first-price or Dutch auction. Then the value $\alpha = 1/\rho - 1$ in an auction with spiteful bidders induces an identical symmetric equilibrium bidding strategy, regardless of n and* *the distribution of valuations.*

Thus the case $n = 2$ and $\alpha = 1$ corresponds to $\rho = 1/2$, which matches closely the value of 0.47 obtained experimentally in first-price, two bidder auction experiments by Holt and Sherman (2001), who observe that this is "essentially 'square root' utility". (They measure an intercept not significantly different from zero, and a slope of 0.667 for the bidding function, exactly as predicted by equilibria found in Subsection 3.1.)

More broadly, Proposition 4 implies that first-price auctions viewed on their own can never offer a way to distinguish the two models. Of course, studying behavior in second price auctions does provide a testable difference between motives of risk-aversion and spite. In the next sections, we suggest a variety of additional implications that *can* be used to discriminate between risk aversion and spiteful incentives—even in first-price settings.

4.2 Interpersonal comparisons

One fundamental difference between our model and risk aversion regards interpersonal comparisons. The heart of our spite motive is that a bidder is making interpersonal comparisons between herself and a representative rival bidder and that these comparisons affect her utility. In the risk averse model, the outcomes of rival bidders do not affect utility at all. Thus, one way to distinguish between the two theories is to vary the degree to which interpersonal comparisons are salient.

Consider two first-price auction treatments. In treatment A, bidders compete in the usual way, against other humans. In treatment B, bidders compete solely against machines (programmed to mimic exactly the bidding strategy of a human population in treatment A) and this fact is known to them. The risk aversion hypothesis predicts no difference in bidding behavior between the two treatments. This is not the case in our model. The interpersonal element is less important in treatment B than in treatment A; thus, one would expect that the "activation" of the α parameter in our model would be lower. As a consequence, we would predict significantly less overbidding in treatment B compared to treatment A.

An alternative version of the same idea is following: In treatment A, subjects are randomly divided into two groups. One group has no control over their bids. They just sit passively and bid 50% of value, the equilibrium bid in first-price auctions when no spite motive is present. The others, call them *active*, get to choose bids. In each round an active is matched against a passive. Both types of bidders are paid at the end of the experiment. All of the above is known to all subjects. In treatment B, there are only active subjects. These are each matched with a computer that is programmed to bid half the value. Again, subjects are paid at the end of the experiment. All of the above is known to all subjects. Once again, the risk aversion hypothesis predicts no difference in bidding between the two treatments. Our model predicts less aggressive bidding in treatment A compared to treatment B.

In our model, higher values of the spite parameter, α, correlate with more aggressive bidding in first and second-price auctions (see Section A.2). In the risk aversion model, overbidding in first-price auctions is uncorrelated with behavior in second-price auctions. Thus, another way to distinguish the two theories would be to have the same set of subjects participate in both types of auctions and study the correlation on a subject basis between overbidding in first-price auctions and overbidding in second-price auctions, and to test the null hypothesis of no correlation against the one-sided alternative suggested by our model.

4.3 Varying the number of bidders

Behavior with respect to variation in the number of bidders remains analogous in both first-price and second-price auctions. We have shown the two models equivalent for first-price auctions. In second-price auctions, the model with spiteful behavior predicts overbidding instead of truthful bidding for risk aversion, but the predicted overbidding is not a function of n. Thus, varying the number of bidders offers no additional discrimination in comparing first-price and second-price auctions.

However, the English auction does offer another way to distinguish between risk aversion and the spite motive. In English auctions with spiteful bidders, the fraction of bidders bidding sincerely increases with n. Thus, with the spite motive, one should see mostly truthful bidding in English auctions until only two bidders remain, followed by a bidding "war" with overbidding between these last two bidders. With risk aversion, we should see only the dominant strategy, truthful bidding, even in the final competition between two bidders.

4.4 Revenue ranking

In the standard case, the Revenue Equivalence Theorem (Myerson, 1981) shows that the revenues of the auction forms we study are all equal. When spite motives are present, the second-price and English auctions remain revenue equivalent, although we expect lower bidding in the English before the point at which the final two bidders compete. Similarly, The Dutch and first-price auctions remain strategically equivalent. But first-price and second-price auctions cease to be revenue equivalent when spite motives are present.

We next consider the revenue ranking between first and second-price auctions where spite motives are present. When valuations are uniformly distributed, the required expected revenues can be evaluated explicitly in closed form. Since this case is the most likely to be used in experiments, we give the results explicitly. The expected revenue in the first-price case is

$$R_{fp} = \frac{n(n-1)(1+\alpha)}{[n+1][n+\alpha(n-1)]}, \tag{29}$$

and the second-price revenue is

$$R_{sp} = \frac{n-1+2\alpha n}{(n+1)(1+2\alpha)}. \tag{30}$$

The difference is

$$R_{sp} - R_{fp} = \frac{\alpha}{[n(1+\alpha)-\alpha][1+2\alpha]} > 0, \tag{31}$$

when $\alpha > 0$. This is a decreasing function of n and α, and is zero when $\alpha = 0$, checking the Revenue Equivalence Theorem. This establishes the ranking Second > First in the uniform case.

As mentioned in Section 2, we can borrow the proof of revenue ranking from earlier work on different models. In particular, Engelbrecht-Wiggans (1994) and Bulow, Huang, and Klemperer (1999) give rankings for some special cases, and Goeree and Turner (2000) give what we believe the first general proof (see Appendix A.4). In summary, we have the following ranking:

Proposition 5 *For general value distributions, all* $0 < \alpha \leq 1$*, and all* $n \geq 2$*, the expected revenue in auctions with spiteful bidders is*

$$\text{English} = \text{Second} > \text{First} = \text{Dutch}. \tag{32}$$

Proposition 5 thus offers another way to distinguish between our model and the alternative of risk aversion. Under risk aversion, the revenue ranking is the opposite of the one we obtain. Thus, a one-sided test where one rejects the null hypothesis of revenue equivalence of first and second-price auctions against the alternative that second-price auctions outperform first-price auctions would also constitute a rejection of the risk aversion model against our model.

4.5 Love of winning

A straightforward alternative model that can generate overbidding in first and second-price auctions is a model where bidders derive utility simply from the act of winning itself. That is, in addition to valuing the object at v, a bidder receives the added utility, w, simply by virtue of winning the auction. This model has the effect of shifting up all values conditional on winning. The upshot is to shift up bidding strategies in both first and second-price auctions by a constant amount compared to the standard model.

This model, however, is easily distinguished from our model by comparing the divergence of predictions from the standard model as a function of a bidder's valuation for the object. In the love-of-winning model, the divergence is independent of valuations; whereas in our model it is not. Specifically, in a first-price auction our model predicts that the divergence between actual bids and the standard prediction increases in a bidder's valuation, while in a second-price auction, this same divergence is predicted to decrease as a bidder's valuation gets higher.

Holt and Sherman run a regression of the form

$$b(v) = \beta_0 + \beta_1 v + \varepsilon \tag{33}$$

to estimate bids in two-bidder first-price auctions where bidders have valuations distributed uniformly. Under the love-of-winning hypothesis, β_0 is predicted to be strictly positive and β_1 is predicted to be $1/2$. In contrast, our model predicts $\beta_0 = 0$ and $\beta_1 > 1/2$. The parameter estimates obtained by Holt and Sherman are consistent with our model and not with the alternative love-of-winning model. More generally, an odd prediction of the love-of-winning model is that bidders with low valuations in first-price auctions will bid in excess of their value for the object. We are unaware of any studies where this prediction is borne out.

5 Conclusions and discussion

We view the main contribution of this paper as pointing out how the spite motive provides a unified explanation for the variety of discrepancies between theory and laboratory data in auction settings.

We have shown that bidders bid more aggressively when they have spiteful incentives. Indeed, equilibrium bids in the second-price auction *exceed* a bidder's valuation for the object, since the price paid by a rival bidder may be determined by a bidder's own bid in this auction. Thus, the usual arguments for weak dominance of sincere bidding do not apply. Finally, when at least two rivals are still active in an English auction (at a given price) a bidder can be sure that by dropping out she will not affect the final price paid. In that case, dropping out at one's own value is an equilibrium strategy for all but the last two bidders. Nonetheless, the final two bidders compete aggressively, knowing that by dropping out they are determining the price paid by a rival bidder. For these reasons, we can expect lower average bidding in English auctions than in second-price auctions, up to the point that only two bidders remain active.

We obtain a revenue ranking: English equals second-price outperforms first-price equals Dutch. This revenue ranking is exactly the opposite to that predicted by risk aversion. In the case of the first-price auction, the equilibrium bidding strategy is behaviorally equivalent to a model where all bidders have preferences exhibiting constant relative risk aversion.

This is not to say that we think spite motives are the only ones present. There is considerable evidence supporting the notion that bidders exhibit risk aversion (see Goeree, Holt and Palfrey, 2000), and structural estimation based on risk aversion has proven highly successful. Nonetheless, if one is restricted to the addition of only one degree of freedom to the standard behavioral model, the spite motive better explains the constellation of stylized facts about subject play in auctions (Anomalies 1-3 in our Introduction).

Taken together, risk aversion and spiteful incentives suggest that additional care is required in inferring valuations of bidders in structural estimation of empirical auction data. Even in a simple model where two bidders compete in a first-price auction, where the spite parameter (α) is unity, and valuations are distributed uniformly, inferring valuations under the assumption that bidders are employing the standard risk-neutral Nash equilibrium strategy leads to upward biased estimates of bidder valuation. Thus, subsequent analysis of optimal reserve prices and so on would necessarily suffer from this bias.

Our model readily lends itself to testing. For example, our model predicts lower average bidding in te English auction. Our model also predicts that overbidding, relative to the standard risk-neutral Nash equilibrium will vary inversely with valuations in the second-price auction, but proportionately with valuations in the first-price auction. This implication may be readily tested. Further afield, our model may be applied to first and second-price all-pay auctions and predicted discrepancies between these equilibria and those in the standard case may be explained. All of this remains for future research.

We conclude by highlighting some of the strengths and limitations of our analysis. First, we have assumed a specific, additively separable functional form for the payoff that models spiteful incentives. Conclusions that these preferences lead to more aggressive bidding in the canonical auction forms in fact hold for a variety of functional forms. Moreover, the arguments that the final two bidders in an English auction bid more aggressively than all preceding bidders also seems quite general. One may be more skeptical of the generality of the revenue ranking conclusions. However, one variant of the models we explored, where a bidder only cares about one particular, representative rival still leads to the conclusions that second-price auctions outperform first-price auctions.

A key limitation of the analysis is the symmetry assumption. While we suspect that overbidding persists when bidders are heterogeneous in the degree of their spiteful incentives, our analysis relies on the symmetry assumption at several junctures. Likewise, we have restricted attention to the simple independent private values case. In many auctions, this assumption is simply not justified, and it remains to be seen what spite motives do to bidding behavior in (say) common value auctions. These extensions remain for future work.

A Appendix

A.1 Proof of global optimality

Consider the expected payoff of bidder 1 with valuation v who bids as though her valuation were x. Without loss of generality, we take bidder 2 to be the winner in the event that 1 is not the high bidder. In a first-price auction, her expected payoff is

$$\mathrm{E}\left(\Delta\right) = \left[v - b(x)\right] G(x) - \alpha(n-1)\mathrm{E}\left(\text{surplus of } 2\right), \tag{34}$$

where $G(x) = F(x)^{n-1}$, the distribution of the highest of $n-1$ independent draws from F (and also the probability that bidder 1 wins). In the second-price auction, the analogous expression is

$$\mathrm{E}\left(\Delta\right) = \int_0^x \left[v - b(t)\right] dG(t) - \alpha(n-1)\mathrm{E}\left(\text{surplus of } 2\right), \tag{35}$$

In both cases, the first derivative of $\mathrm{E}\left(\Delta(x)\right)$ can be written in the following form:

$$\frac{\partial \mathrm{E}\left(\Delta\right)}{\partial x} = vg(x) + b'(x)\Gamma(x,\alpha) + b(x)\Theta(x,\alpha) + \Lambda(x,\alpha). \tag{36}$$

Setting $x = v$ in this equation yields a first-order ordinary differential equation that determines the equilibrium bidding function $b_0(v)$, and it is easy to check that this procedure yields the same differential equations that led to the equilibria given in the text. Substituting $b_0'(x)$ from this differential equation back into Eq. (36) yields the following simple expression for the first partial derivative of payoff with respect to the reported x using the equilibrium strategy:

$$\frac{\partial \mathrm{E}\left(\Delta\right)}{\partial x} = (v-x)g(x), \tag{37}$$

Now clearly, this expression is positive for $x < v$ and negative for $x > v$; therefore, bidder 1 can do no better than to choose $x = v$. Thus, we have shown that no profitable deviations are possible.

To see the natural economic interpretation of Eq. (37), expand the expected payoff in a power series about $x = v$:

$$\mathrm{E}\left(\Delta\right)(x) \approx \mathrm{E}\left(\Delta\right)(v) - \frac{g(v)}{2}(x-v)^2, \tag{38}$$

where we used the fact that the first derivative vanishes at $x = v$. From this we see that reporting a value to the third party in the revelation game that differs from the truth, v, by δv results in a decrease in payoff that is to first order $-(g(v)/2)\delta v^2$, proportional to the probability density of the largest value of the rival bidders at the valuation v of bidder 1. At a valuation where one is likely to find the highest-valued rival, there is a high price to pay for deviating from truthful reporting, and vice versa.

Equation (36) shows the explicit dependence on the bidding function b in order to make it clear how the first order necessary condition leads to an ordinary differential equation for b; this is an alternate method of deriving the equilibria in the text. But the argument is quite general, and depends only on v appearing in the expected payoff of the revelation mechanism for an auction *only* as the expected value received, $vG(x)$, where $G(x)$ is the probability of winning with reported value x. In these cases the first partial of the expected payoff is of the form

$$\frac{\partial \mathrm{E}\left(\Delta\right)}{\partial x} = vg(x) + \Psi(x), \tag{39}$$

where we have used the putative equilibrium bidding function $b(x)$ for bidder 1, and taken expectations with respect to the values v_i of her rivals, who also, symmetrically, use $b(v_i)$. Setting this equal to zero when $x = v$, replacing v with x everywhere to get functions of x, and subtracting yields the same result as Eq. (37). Thus the proof of sufficiency for optimality applies to a wide class of auctions, including all-pay and other variations where rivals' surplus is included in the payoff function.

A.2 Proof of bid monotonicity in n and α

We show that the equilibrium bidding functions in first-price auctions where spite motives are present are monotonically increasing functions of n and α. Let $k = (n-1)(1+\alpha)$, fix v, and write Eq. (24) as

$$b(k) = v - \frac{\int_0^v F(t)^k dt}{F(v)^k}. \tag{40}$$

Now consider the effect of increasing k by δ:

$$\begin{aligned} b(k+\delta) - b(k) &= -\frac{\int_0^v F(t)^{k+\delta} dt}{F(v)^{k+\delta}} + \frac{\int_0^v F(t)^k dt}{F(v)^k} \\ &= \frac{-\int_0^v F(t)^{k+\delta} dt + F(v)^\delta \int_0^v F(t)^k dt}{F(v)^{k+\delta}}. \end{aligned} \tag{41}$$

Notice that since F is increasing, the factor $F^\delta(t)$ in the first integrand can be upper bounded by its value at the right limit, v, which implies

$$-\int_0^v F(t)^{k+\delta} dt > -F(v)^\delta \int_0^v F(t)^k dt, \tag{42}$$

which in turn implies that $b(k+\delta) - b(k) > 0$.

Almost exactly the same argument shows that the bidding function in the second-price case, Eq. (27), is a monotonically decreasing function of $m = (1+\alpha)/\alpha$, and hence increasing in α.

A.3 Derivation of the first-price Constant Relative Risk Averse (CRRA) equilibrium

We include a derivation of the risk-averse equilibrium for the first-price case, after Riley and Samuelson (1981). The expected payoff of bidder 1, using the absolute profit criterion and utility function $u(t)$, is

$$\mathrm{E}\,(\,\text{payoff}\,) = u(v - b(v)) F(\beta^{-1}(b(v)))^{n-1}, \tag{43}$$

where bidder 1 with valuation v must choose her bid $b(v)$, other bidders i bid $\beta(v_i)$, and the values are independent and identically distributed as F. Differentiating with respect to b and setting $\beta = b$ gives the first-order condition for a symmetric equilibrium,

$$b'(v) = (n-1)\frac{u(v-b(v))}{u'(v-b(v))}\frac{f(v)}{F(v)}. \tag{44}$$

In the CRRA case, defined by the utility function $u(t) = t^\rho$, $u(t)/u'(t) = t/\rho$, and this becomes the same equation as in the risk-neutral case, but with $(n-1)$ replaced by

$(n-1)/\rho$. The solution is therefore the absolute profit result, Eq. (24), with F-exponents $(n-1)/\rho$ instead of $(n-1)(1+\alpha)$:

$$b(v) = v - \frac{\int_0^v F(t)^{(n-1)/\rho} dt}{F(v)^{(n-1)/\rho}}. \tag{45}$$

It follows that the equilibrium bidding strategy in the first-price, CRRA, absolute profit case is precisely the same as that in the first-price, risk-neutral, spiteful incentive case, with correspondence $\rho = 1/(1+\alpha)$.

A.4 Proof of revenue ranking Second > First

We adapt the proof due to Goeree and Turner (2000) to our model. Their proof is, in turn, a generalization of earlier proofs by Engelbrecht-Wiggans (1994) and Bulow, Huang, and Klemperer (1999). We give the details here for completeness. We begin by putting the equilibrium bidding functions in the form of conditional expectations. In the first-price case we have

$$b_1(x) = \int_0^x s dF_1(s|x)\,, \tag{46}$$

where

$$F_1(s|x) = \left(\frac{F(s)}{F(x)}\right)^{(n-1)(1+\alpha)}, \tag{47}$$

and in the second-price case,

$$b_2(x) = \int_x^1 s dF_2(s|x)\,, \tag{48}$$

where

$$F_2(s|x) = 1 - \left(\frac{1-F(s)}{1-F(x)}\right)^m, \tag{49}$$

and $m = (1+\alpha)/\alpha$.

The total revenues are

$$R_i = \int_0^1 b_i(x) dH_i(x) \tag{50}$$

where H_i is the distribution of the appropriate order statistic,

$$H_i(x) = \begin{cases} F(x)^n & i = 1 \text{ (first price)} \\ nF(x)^{n-1} - (n-1)F(x)^n & i = 2 \text{ (second price)}, \end{cases} \tag{51}$$

the first being the distribution of the largest of n values, and the second the distribution of the second-largest of n values. After some algebra, the corresponding revenues are[8]

$$R_1 = \frac{n(n-1)(1+\alpha)}{1-\alpha(n-1)} \int_0^1 x \left[1 - F(x)^{1-\alpha(n-1)}\right] F(x)^{(n-1)(1+\alpha)-1} dF(x)\,, \tag{52}$$

and

$$R_2 = mn(n-1) \int_0^1 x(1-F(x))^{m-1} \int_0^x \frac{F(y)^{n-2}}{[1-F(y)]^{m-1}}\, dF(y)\; dF(x)\,. \tag{53}$$

[8]Note that the case $\alpha = 1$ is special, but can be handled by arguing that the revenue functions are continuous functions of α.

With the one-to-one transformation $t = F(x)$, R_2 can be written

$$R_2 = mn(n-1)\int_0^1 F^{-1}(t)(1-t)^{m-1}\int_0^{F^{-1}(t)} \frac{F(y)^{n-2}}{[1-F(y)]^{m-1}}\, dF(y)\; dt\,. \tag{54}$$

Performing the same transformation, $z = F(y)$, in the inner integral yields

$$R_2 = mn(n-1)\int_0^1 F^{-1}(t)(1-t)^{m-1}\int_0^{t} \frac{z^{n-2}}{(1-z)^{m-1}}\, dz\; dt\,. \tag{55}$$

The difference in revenues between second and first-price auctions can then be put in the form

$$\Delta R = R_2 - R_1 = \int_0^1 F^{-1}(t)\delta(t)\; dt\,, \tag{56}$$

where $\delta(t) = \delta_2(t) - \delta_1(t)$,

$$\delta_1(t) = \frac{n(n-1)(1+\alpha)}{1-\alpha(n-1)}(1 - t^{1-\alpha(n-1)})t^{(n-1)(1+\alpha)-1}\,, \tag{57}$$

and

$$\delta_2(t) = mn(n-1)^2\int_0^t \left(\frac{1-t}{1-z}\right)^{1/\alpha} z^{n-2}\; dz \tag{58}$$

are the components of $\delta(t)$ corresponding to the first and second-price auctions, respectively. Note that $F^{-1}(t)$ is an increasing function.

The following facts can be checked in a straightforward manner:

(a) The integrals of $\delta_i(t)$ over the unit interval are both unity:

$$\int_0^1 \delta_1(t)\; dt = \int_0^1 \delta_2(t)\; dt = 1\,, \tag{59}$$

so that the integral of $\delta(t) = \delta_2(t) - \delta_1(t)$ over the unit interval is zero:

$$\int_0^1 \delta(t)\; dt = 0\,. \tag{60}$$

(b) $\delta(0) = 0$, and for small t, $\delta(t) < 0$. (Expand $\delta(t)$ in a power series in t.)

(c) The derivative of $\delta(t)$ at zero crossings can be expressed as

$$\delta'(t)\Big|_{\delta(t)=0}\left[\frac{\alpha(1-t)}{t^{(n-1)(1+\alpha)-2}}\right] = \frac{t^{1-\alpha(n-1)} - t + \beta(t-1)}{1-\alpha(n-1)} \stackrel{\text{def}}{=} h(t)\,, \tag{61}$$

where $\beta = \alpha\left[(n-1)(1+\alpha)-1\right] > 0$. Therefore the sign of $\delta'(t)$ at zero crossings of $\delta(t)$ is the same as the sign of $h(t)$.

(d) $h(0+) < 0$ and $h(1) = 0$.

(e) The function $h(t)$ is concave on [0, 1]; that is, $h''(t) \le 0$.

Putting these facts together: $\delta(t)$ begins at $t = 0$ by going negative. It reaches zero again at $t = 1$, and its integral on $[0, 1]$ is zero, so it must become positive at some point in $(0, 1)$. Let t^* be the first such point. At this zero crossing, its derivative is positive, so the sign of $h(t^*)$ is positive.

The function $h(t)$ begins negative at $t = 0$, and becomes positive by the time $t = t^*$. It is concave and reaches zero at $t = 1$. It therefore cannot dip below zero in $(t^*, 1)$, for otherwise its curvature would change, contradicting the fact that it is concave. The sign of $\delta'(t)$ at any possible zero crossing in $(t^*, 1)$, which is the same as the sign of $h(t)$, must therefore be positive, which shows that $\delta(t)$ remains positive on $(t^*, 1)$.

Thus, $\delta(t)$ is negative on $(0, t^*)$ and positive on $(t^*, 1)$. From Eq. (56), the revenue difference between second and first price auctions, ΔR, is the integral on the unit interval of an increasing function times $\delta(t)$. Because $\delta(t)$ is negative and then positive, without further sign changes, and the areas of the two pieces of $\delta(t)$ are equal, $\Delta R \geq 0$. ■

References

Bikhchandani, S., P. Haile, and J. Riley, "Symmetric Separating Equilibria in English Auctions," *Games and Economic Behavior*, 2001 (forthcoming).

Bishop, D. T., C. Canning and J. Maynard Smith, "The War of Attrition with Random Rewards," *Journal of Theoretical Biology* **74**, (1978), pp. 377–388.

Bolton, G. E., and A. Ockenfels, "ERC — A Theory of Equity, Reciprocity, and Competition," *American Economic Review*, forthcoming.

Bulow, J., M. Huang, and P. Klemperer, "Toeholds and Takeovers," *J. Political Economy* **107**, 3 (1999), pp. 427–454.

Bulow, J. and P. Klemperer, "Rational Frenzies and Crashes," *J. Political Economy*, **102**, 1, 1994, pp. 1–23.

Cassady, Jr., R., *Auctions and Auctioneering,* University of California Press, Berkeley, 1967.

Cox, J. C., B. Roberson, and V. L. Smith, "Theory and Behavior of Single Object Auctions," in *Research in Experimental Economics* (V. L. Smith, ed.), JAI Press, Greenwich, Conn. (1982).

Das Varma, G., "Standard Auctions with Identity Dependent Externalities," RAND Journal of Economics, (forthcoming).

Davis, D. D. and C. A. Holt, *Experimental Economics*, Princeton University Press, Princeton, N.J., 1993.

Engelbrecht-Wiggans, R., "Auctions with Price-Proportional Benefits to Bidders," *Games and Economic Behavior*, **6** (1994), pp. 339–346.

Engers, M., and B. McManus, "Charity Auctions," working paper, November 2001.

Ettinger, D., "Auctions and Shareholdings," working paper, CREST-LEI and CERAS-ENPC, Paris, November, 2002.

Fehr, E. and K. M. Schmidt, "A Theory of Fairness, Competition, and Cooperation," *Quarterly Journal of Economics* CXIV (1999), pp. 817–868.

Goeree, J. K., C. A. Holt, T. R. Palfrey, "Quantal Response Equilibrium and Overbidding in Private Value Auctions," working paper, February 2000.

Goeree, J. K. and J. L. Turner, "All-Pay-All Auctions," working paper, December 2000.

Hamilton, W. D., "Selection of Selfish and Altruistic Behavior in Some Extreme Models," in *Man and Beast: Comparative Social Behavior* (J. F. Eisenberg and W. S. Dillon, eds.), Smithsonian Institution, Washington D.C. (1971).

Holt, C. A. and R. Sherman, "Risk Aversion and the Winner's Curse," Unpublished manuscript, April, 2000.

Jehiel, P. and B. Moldovanu, "Auctions with Downstream Interaction Among Buyers," *RAND Journal of Economics,* **31**, (2000), pp. 768-791.

Jehiel, P., B. Moldovanu, and E. Stacchetti, "How (Not) to Sell Nuclear Weapons," *American Economic Review,* **86**, (1996), pp. 84-98.

Kagel, J. H., "Auctions: a survey of Experimental Research," in *The Handbook of Experimental Economics*, (J. H. Kagel and A. E. Roth, eds.), Princeton University Press, Princeton, NJ, 1995.

Kagel, J., R. Harstad, and D. Levin, "Information Impact and Allocation Rules in Auctions with Affiliated Private Values," *Econometrica,* **55**, 6 (1987), pp. 1275-1304.

Kells, L. M., *Elementary Differential Equations*, McGraw-Hill, 1954.

Koçkesen, E., Ok, and R. Sethi, "The Strategic Advantage of Negatively Interdependent Preferences," *Journal Economic Theory* **92** (2000), pp. 274–299.

Levine, D. K., "Modeling Altruism and Spitefulness in Experiments," *Review of Economic Dynamics*, **1** (1998), pp. 593-622.

Milgrom, P. and R. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica,* **50** (1982), pp. 1089–1122.

Myerson, R., "Optimal Auction Design," *Mathematics of Operations Research,* **6** (1981), pp. 58–73.

Rabin, M., "Incorporating Fairness into Game Theory and Economics," *American Economic Review* LXXXIII (1993), pp. 1281–1302.

Rheims, M., *The Strange Life of Objects: 35 Centuries of Art Collecting,* Anthenum Publishers, New York, 1961, (translated from the French by D. Pryce-Jones).

Riley, J. G. and W. F. Samuelson, "Optimal Auctions," *American Economic Review*, **71**, 3 (1981), pp. 381–392.

Saijo, T. and H. Nakamura, "The 'Spite' Dilemma in Voluntary Contribution Mechanism Experiments," *Journal of Conflict Resolution*, **39**, 3 (1995), pp. 535–560.

Maynard Smith, J., *Evolution and the Theory of Games*, Cambridge University Press, New York, 1982

Vickrey, W., "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, **16**, 1 (1961), pp. 8–37.