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Noise-immune universal computation using Manakov soliton collision cycles

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Abstract: We show that bistable collision cycles of Manakov solitons are capable of universal, all-optical computation with state restoration. NAND gates and FANOUT are realized by soliton collisions in a homogeneous nonlinear medium.
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1. Introduction

It was shown in Ref. [1] that collisions of solitons governed by the Manakov equations in an ideal medium not subject to noise could perform arbitrary computation. However, demonstrating state restoration is fundamental to the practical realization of any noise-immune computing scheme. In this paper, we demonstrate computational universality using recently described collision cycles of Manakov solitons for state restoration [2].

The configuration used in this paper can be realized in any nonlinear optical medium that supports propagation of Manakov solitons. Various media have approximated such propagation, including photorefractives [3-7], semiconductor waveguides [8], quadratic media [9], and optical fiber [10].

This work joins a growing list of alternative computing paradigms, including DNA computing [11], chaos computing [12], quantum computing (see [13], for example), and even ideal billiard balls [14].

2. Framework

Our model is based on the Manakov system, which consists of two coupled nonlinear Schrödinger equations

$$\begin{aligned} i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + 2\mu\left(|u|^2 + |v|^2\right)u = 0, \\ i\frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial x^2} + 2\mu\left(|u|^2 + |v|^2\right)v = 0, \end{aligned} \tag{1}$$

where $u = u(x,t)$ and $v = v(x,t)$ are two interacting optical components, μ is a positive parameter, and x and t are normalized space and time. The two components can be thought of as components in two polarizations, or, as in the case of a photorefractive crystal, two uncorrelated beams. This system admits single-soliton, two-component solutions that can be characterized by a complex number $k = k_R + ik_I$, where k_R determines the energy of the soliton, and k_I is the velocity, all in normalized units. The *polarization state* is represented by a complex number ρ which is the ratio between the u and v components.

It was shown, using explicit solutions of Radhakrishnan *et al.* [15], that collisions of these solitons are described by explicit linear fractional transformations of the complex polarization state [16]. Consider a collision between two

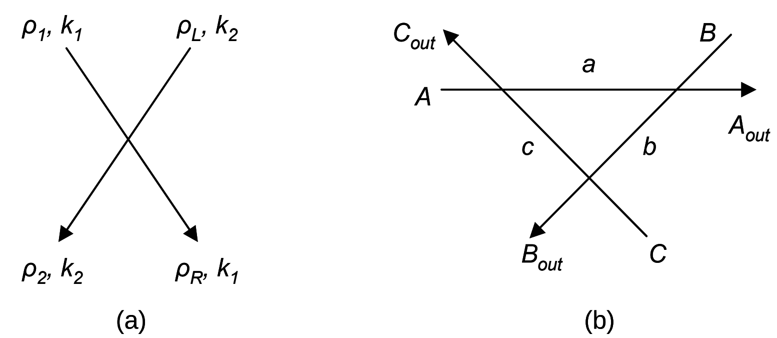

Fig. 1. (a) General two-soliton collision; (b) Schematic of three-soliton collision cycle.

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solitons as shown in Fig. 1(a), where k_1 and k_2 represent the constant soliton parameters, corresponding to the right-moving and left-moving solitons, respectively. Let ρ_1 and ρ_L denote the respective soliton states before impact, and suppose the collision transforms ρ_1 into ρ_R, and ρ_L into ρ_2. Explicitly, the state of the emerging left-moving soliton is given by

$$\rho_2 = \frac{\left[(1-g)/\rho_1^* + \rho_1\right]\rho_L + g\rho_1/\rho_1^*}{g\rho_L + (1-g)\rho_1 + 1/\rho_1^*},\ g = \frac{k_1 + k_1^*}{k_2 + k_1^*},\ \text{and } k_{1R},\ k_{2R} > 0. \tag{2}$$

Using this transformation, a 3-soliton collision cycle, shown schematically in Fig. 1(b), was found to demonstrate bistability [2]. In this schematic, A, B, C, etc. represent complex-valued soliton states ρ. Bistability is found in the steady-state value of the polarization state, and will form the basis for our proof of computational universality. One instance of bistability is shown in Fig. 2, where we follow the same procedure as outlined in Ref. [2] for calculating the two foci and their corresponding basins of attraction. The foci, labeled a_0 and a_1, are the two steady-state values of a, and correspond to the value of that beam in binary state *0* and *1*, respectively. The basins of attraction illustrate those initial values (of state a) that converge to each basin. The boundary between the basins of attraction is a kind of 2-D threshold, analogous to the switching in ordinary transistor-based logic. Note that beams A, B, C, and the values k_1 and k_2 remain constant in this and all subsequent simulations in this paper, while a, b, c, A_{out}, B_{out}, and C_{out} can each have two stable steady-state values, depending on the binary state of the cycle. If A, B, or C is changed, the basins and foci will change, and we can lose bistability altogether, resulting in only one steady-state focus.

3. Proof of Universality

We initialize a cycle by colliding an external control beam with beam A. Upon collision, the cycle is designed to become monostable, and is allowed to reach steady-state, where its focus is known. When the control beam is switched off, the cycle retains its original bistable form, but we now know its initial condition, and hence the state to which it will converge. In this manner, we can individually set or reset a cycle, much like a flip-flop.

To show computational universality via the NAND operation, we define the output of a cycle as B_{out}, and use it to collide with A, as shown in Fig. 3(a). The inputs to the NAND gate are the outputs from two other cycles (not

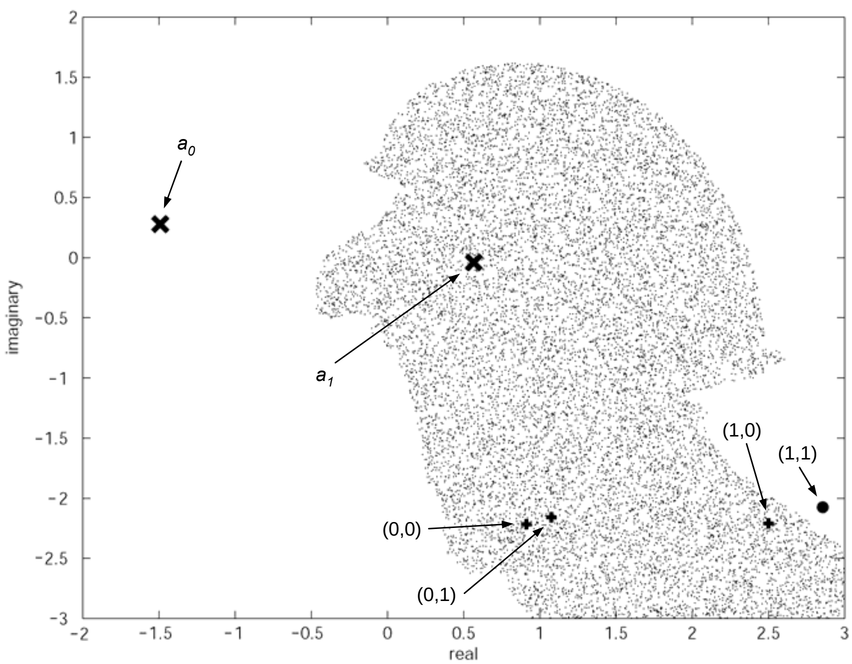

Fig. 2. Map of beam a in the complex plane showing NAND gate operation. The two foci, a_0 and a_1, are shown with their corresponding basins of attraction. The + signs indicate inputs where the cycle reaches state 1, the ● indicates a (1, 1) input. The states of the input beams are $A = -0.2 + 0.2i$, $B = 0.9 + 1.5i$, $C = -0.5 - 1.5i$; and $k_1 = 4 + i$, $k_2 = 4 - i$.

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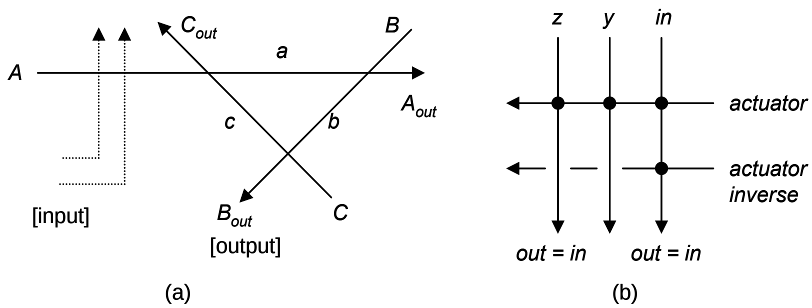

Fig. 3. (a) NAND gate; (b) FANOUT gate, each ● indicates a collision, z = 1.1587 - 0.4428i, y = -0.4553 - 0.2570i, *actuator* = *B*.

shown), both with the same bistable configuration, each of which can be set in binary state *0* or *1*. When these inputs are active, the NAND gate will become monostable. Turning off the inputs will place the cycle in the state corresponding to the NAND operation. These monostable foci are shown in Fig. 2. Only when the inputs are both in state *1* will the cycle be put into state *0*.

The last requirement for universal computation, FANOUT, is shown in Fig. 3(b), where we adopt the design of Ref. [1]. The inverse of a polarization state ρ is defined as -1/ρ*; a collision with the inverse of ρ after a collision with ρ restores the original state. Wire crossings are accomplished by time gating the beams to avoid unwanted collisions, as in [1].

4. Discussion

Ideally, the system of Eq. (1) is reversible, whereas a NAND gate is not. The reason our realization works irreversibly is that we assume a noisy environment—that is, when the polarization states are sufficiently close to a focus, the history of the previous state is lost, much as in a transistor-based flip-flop. Note also that while theoretically, energy is conserved in the system of Eq. (1) and there exists no inherent lower bound on energy dissipation, erasure costs on the order of kT per bit [17], and we expect an analogous cost in our irreversible NAND gate.

We have defined a 3-soliton bistable collision cycle that also serves as a NAND gate when the proper interconnections are made. Each stage of logic is self-restoring, demonstrating the noise immunity of such an all-optical computing scheme.

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