Myerson & Satterthwaite (4)

"Efficient Mechanisms for Bilateral Trading"

Ultimately simple situation:

1 seller, 1 buyer

value $\tilde{V}_1$ value $\tilde{V}_2$

IPV $\tilde{V}_1$ is distributed as density $f_1(\cdot) > 0$ on $[a_1, b_1]$

$\tilde{V}_2$ $f_2(\cdot) > 0$ on $[a_2, b_2]$

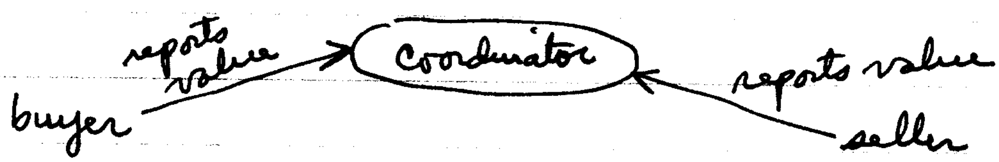
risk neutral

additively separable utility for money and object

Basic Question: Among all possible bargaining mechanisms, which have desirable economic efficiency properties?

Review of Revelation Principle:

A direct bargaining mechanism



- direct mechanism: Characterization (p, x), where

$$p(v_1, v_2) = \text{prob. transfer } 1 \to 2$$
$$x(v_1, v_2) = \text{expected payment } 1 \to 2$$

A direct mechanism is (Bayesian) incentive-compatible if honest reporting forms a BNE.

Revelation Principle

For every equilibrium in a bargaining game, $\exists$ equivalent incentive compatible direct mechanism.

Proof if there were an incentive to lie when reporting then there would be an incentive to lie to oneself in the original game.

$\therefore$ we can restrict attention to incentive-compatible direct mechanisms.

So we have (p, x).
Some basic quantities:

$$\bar{x}_1(v_1) = \int_{a_2}^{b_2} x(v_1, t_2)\, f_2(t_2)\,dt_2 = E[\text{revenue of seller}]$$

$$\bar{p}_1(v_1) = \int_{a_2}^{b_2} p(v_1, t_2)\, f_2(t_2)\,dt_2 = \text{prob. of 1 selling to 2}$$

$$U_1(v_1) = \bar{x}_1(v_1) - v_1 \cdot \bar{p}_1(v_1) = E[\text{profit of seller, 1}]$$

& similarly,

$$U_2(v_2) = v_2 \cdot \bar{p}_2(v_2) - \bar{x}_2(v_2) = E[\text{profit of buyer, 2}]$$

In this notation, incentive-compatible means

$$U_1(v_1) \geq \bar{x}_1(\hat{v}_1) - v_1 \cdot \bar{p}_1(\hat{v}_1) \quad \forall\, v_1, \hat{v}_1 \in [a_1, b_1]$$

$$U_2(v_2) \geq v_2 \cdot \bar{p}(\hat{v}_2) - \bar{x}_2(\hat{v}_2) \quad \forall\, v_2, \hat{v}_2 \in [a_2, b_2]$$

Another desirable property:

A mechanism is individually rational iff

$$U_1(v_1) \geq 0 \qquad \forall v_1 \in [a_1, b_1],\ v_2 \in [a_2, b_2]$$
$$U_2(v_2) \geq 0$$

And another desirable property:

A mechanism is ex post efficient iff

$$p(v_1, v_2) = \begin{cases} 1 & \text{if } v_1 < v_2 \\ 0 & v_1 > v_2 \end{cases}$$

I.e. object is sold iff buyer values it more highly.

Main Result (Corollary 1)

If $[a_1, b_1] \cap [a_2, b_2] \neq \emptyset$ (overlap)

Then

no incentive-compatible individually rational trading mechanism can be ex post efficient!

Some details of proof technique:

first part (Theorem 1)

incentive-compatible
& individually rational $\Longrightarrow$ $U_1(b_1) + U(a_2) \geq 0$

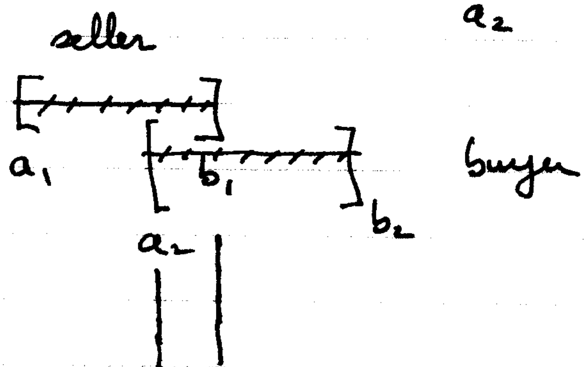
$\uparrow$ min. expected profit of seller
$\uparrow$ min. expected profit of buyer

follows from definitions and crank turning.

Second part

ex post efficient $\Longrightarrow$ $U_1(b_1) + U_2(a_2)$

$$= -\int_{a_2}^{b_1} [1 - F_2(t)]\, F_1(t)\, dt < 0$$



This quantity, $\int_{a_2}^{b_1} [1 - F_2(t)]\, F_1(t)\, dt$ is the smallest lump-sum subsidy to create a Mechanism that is

- Incentive compatible
- individually rational
- ex post efficient

[cf the beginning of Vickrey 61!]

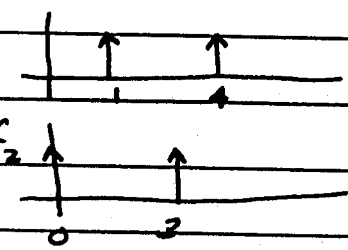
Example 1 Shows that it is necessary that $f_i > 0$ on respective intervals.

Counterexample Suppose

$$\text{prob}(\tilde{V}_1 = 1) = \text{prob}(\tilde{V}_1 = 4) = \text{prob}(\tilde{V}_2 = 0) = \text{prob}(\tilde{V}_2 = 3) = 1/2$$

seller f_1

buyer f_2



seller \ buyer 2	0	3
1	1/4	1/4 ← profitable to both
4	1/4	1/4
	0	3

Claim: the mechanism "sell at price 2 if both are willing, else no trade"
is incentive-compatible, individually rational, & ex post efficient.

- incentive-compatible honest reporting is a BNE (check)
- individually rational clearly E[profit] > 0
- efficient trade occurs when & only when $v_1 < v_2$.

<u>Example 2</u> $\tilde{v}_1, \tilde{v}_2$ both Uniform $[0,1]$

Theorem becomes

$$U_1(b_1) + U_2(a_2) = \int_{a_2}^{b_2}\int_{a_1}^{b_1} \left[v_2 - \frac{1-F_2(v_2)}{f_2(v_2)}\right] - \left[v_1 + \frac{F_1(v_1)}{f_1(v_1)}\right]$$

$$\cdot\, p(v_1, v_2) \cdot f_1(v_1) f_2(v_2)\, dv_1 dv_2$$

$$= \int_0^1\int_0^1 \left\{\left[v_2 - \frac{1-v_2}{1}\right] - \left[v_1 + \frac{v_1}{1}\right]\right\} p(v_1, v_2)\, dv_1 dv_2$$

or

$$\int_0^1\int_0^1 \left(v_2 - v_1 - \frac{1}{2}\right) p(v_1, v_2)\, dv_1 dv_2 \geq 0$$

or

$$\int_0^1\int_0^1 (v_2 - v_1)\, p(v_1, v_2)\, dv_1 dv_2 - \frac{1}{2}\int_0^1\int_0^1 p(v_1, v_2)\, dv_1 dv_2 \geq 0$$

divide $\rightarrow$ conditional prob.

or

$$E\left[v_2 - v_1 \mid \text{trade}\right] \geq \frac{1}{2}.$$

But in general

$$E\left[v_2 - v_1 \mid v_2 \geq v_1\right] = 1/3$$

$E[\text{subsidy}]$ required for ex post efficiency $= 1/6$.

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check: (7) of M&S 83 $\displaystyle\int_{a_2}^{b_1} [1 - F_2(t)]\, F_1(t)\, dt$

$$= \int_0^1 (1-t)\, t\, dt = 1/6 \quad \checkmark$$
